Lab 2: AC Measurements—Capacitors and Inductors

Introduction

The second most common component after resistors in electronic circuits is the capacitor. It is a two-terminal device that stores an electric field when charged. Two electrodes inside the capacitor, separated by a small distance, hold positive charges on one electrode and negative charges on the other. An electrical field or voltage is developed across the electrodes. Unlike resistors, which require only one parameter to describe their AC circuit properties, a capacitor requires three:

1. Magnitude \( C \) (measured in farads)
2. Frequency \( f \) (measured in Hertz or radians)
3. Phase \( \phi \) (degrees or radians)

Purpose

This lab discovers these parameters and demonstrates how you can visualize and measure them. National Instruments myDAQ tools for AC signal generation and measurements, which include the function generator, oscilloscope, and Bode analyzer, are introduced.

Equipment

- NI myDAQ
- 1 k\( \Omega \) Resistor
- 0.1 \( \mu \)F Capacitor
- Solderless Breadboard (recommended)

Prerequisite Reference Materials

How to use the NI ELVISmx Function Generator (FGEN):
http://decibel.ni.com/content/docs/DOC-12940

How to use the NI ELVISmx Oscilloscope (Scope):
http://decibel.ni.com/content/docs/DOC-12942

How to use the NI ELVISmx Bode Analyzer (Bode):
http://decibel.ni.com/content/docs/DOC-12943
Exercise 2-1: Getting Started

The first step is to build a simple RC circuit (a circuit with a resistor and capacitor).

![Fig. 1 RC Voltage Divider Circuit](image)

**Note:** NI myDAQ is uniquely suited for this measurement because the negative analog input (AI #) perform differential measurements, meaning they are not directly tied to ground (AGND).

Connect a 1 kΩ resistor to the function generator (FGEN) output (AO 0) screw terminal on the side on your myDAQ connector block. A 0.1 μF capacitor is connected in series with the resistor. The other end of the capacitor goes to the AGND screw terminal. The capacitor voltage is measured with two leads across the capacitor and is connected to the AI 1+ and AI 1- screw terminals. The resistor voltage is measured with two leads across the resistor and is connected to the AI 0+ and AI 0- screw terminals.

From your NI ELVISmx Instrument Launcher strip, select [FGEN].
Configure the FGEN as a sine wave generator shown in Fig. 2.

*Note: The only output of the function generator is via the analog output screw terminals labeled AO 0 and AGND on the side connector strip.*

From your NI ELVISmx Instrument Launcher strip, select [Scope].
Verify your connections and settings (Fig. 3), and then press the [Run] button in the ‘Run Continuously’ Acquisition Mode on both the FGEN and Scope instruments.

You are now looking at the voltage drop across the resistor (green trace). Verify that the frequency measured on the Scope is the same as that set on the FGEN.

Now by clicking on the [Enabled] boxes, disable the green trace and enable the blue trace.

You are now looking at the voltage across the capacitor (blue trace). It is a sine wave of the same frequency but with a smaller amplitude than the resistor voltage.

However, there is another big difference. Let’s discover it.

Try different frequencies on the FGEN. What happens to the capacitor voltage?

As the frequency is changed, the signal amplitude of the voltage across the capacitor also changes. This shows that the capacitor voltage
depends not only on the magnitude of the capacitor but also on the frequency (the larger the frequency, the smaller the amplitude). In fact, the capacitor voltage depends on the inverse of the frequency \(1/f\).

Return to the original frequency of 1.0 kHz.

Now enable the green trace so both traces are seen. Both traces are sine waves of the same frequency but with different amplitudes. Note also that the capacitor voltage (blue trace) lags or follows the resistor voltage (green trace). If one was to measure the position of the blue peak with respect to the green peak, then this lag can be interpreted as a minus time difference.

Click the [Cursors On] box. Two cursors (C1 and C2) show up on the left side on the graph. Click and drag each one in turn to rest on successive peaks: one on a blue and one on a green peak (Fig. 4).

![Fig. 4 The R and C Circuit Waveforms](image)

Record the time difference \(dT\) between the two peaks as a measure of the phase shift.
Lab 2. AC Measurements

\[ dT = \text{__________} \]

Now move one of the cursors to a successive peak on the same color. This measurement of dT gives the period T. Record your measurement.

\[ T = \text{__________} \]

The phase \( q \) between the two signals is calculated from the ratios:

\[ \frac{dT}{T} = \frac{q}{360} \]

What a surprise! It is just 90 degrees. Now adding the lag property (sign is negative), then the phase difference is \( -90 \) degrees or \( -\pi/2 \) radians. In complex notation, this phase difference is given by \( -(j) \) or \( (1/j) \).

Capacitors have only an imaginary \( (j) \) component.

For AC circuits, a capacitor behaves like a resistor and is called the capacitive reactance \( X_C \). Its magnitude is given by \( (1/wC) \), and its phase is \( -90 \) degrees or \( (1/j) \).

Reactance of a capacitor is a vector \( X_C = (1/jwC) \).

**Inductance**

If one was to replace the capacitor in our circuit with an inductor and follow the same procedure as above, then one would discover that inductance is also imaginary. An inductor in an AC circuit behaves like a resistor and is called inductive reactance \( X_L \).

Inductors have the following:
- Magnitude, \( L \) (measured in Henries)
- Frequency, \( w \) (measured in radians)

and
- Phase, \( j \) (inductance signal leads the resistance signal)

The reactance of an inductor is a vector \( X_L = (jwL) \).

While a charged capacitor stores an electric field, an inductor stores a magnetic field. Electronics is just applied E&M (electro-magnetic) theory.
Voltage Divider

Any circuit with a combination of resistors, capacitors, and inductors can be replaced with a single component called impedance. It is a vector with both real and imaginary parts and is written as

\[ Z = R + X_c + X_L \]

It can be visualized as a vector with a real part (R) plotted on the X-axis and an imaginary part \( \{X_c, X_L\} \) plotted on the Y-axis.

The vector sum of all the components (real and imaginary) is a vector of magnitude \(|Z|\) and phase angle \(q\), where \( q = \arctan(\text{imaginary part/real part}) \).

The simplest electronic circuit is a voltage divider, like our RC circuit. It can be represented by the following vector diagram.

![Fig. 5 Phasor or Vector Diagram of a Complex RC Impedance](image)

Exercise 2-2: Bode Plots

One of the best ways to visualize the frequency dependence of an impedance circuit is with a Bode plot. Here one plots the voltage gain (dB) versus the log of the frequency to reveal a characteristic curve for the real part (magnitude) of a circuit. A second plot of the phase (linear scale) versus the log of the frequency shows a characteristic curve for the imaginary part of the circuit. NI myDAQ automates this measurement with a Bode Analyzer (BODE).

From your NI ELVISmx Instrument Launcher strip, select [BODE].
Verify the settings shown in Fig. 6.

**Notes:**

*Stimulus Channel [AI 0]* from FGEN to myDAQ screw terminals (FGEN to AI 0+, AGND to AI 0-).

*Response Channel [AI 1]* from capacitor voltage to screw terminals (Capacitor voltage to AI 1+, AGND to AI 1-).

Graph ranges have been changed for (Auto) to a Gain setting (0 to −16 db) and a Phase setting (0 to −90 degrees).

Press [Run] to view the Bode plot.

You can immediately see the entire frequency response for the RC network. The voltage gain is a constant until the reactance of the capacitor becomes significant. Then the gain falls off as a straight line for $f > 2$ kHz.

On a log-log plot such as in the gain curve, a straight line indicates a power law. Here the power is $-1$ or $1/f$.

You can see that the phase of the circuit is resistive (0 degrees) for small frequencies but capacitive (−90 degrees) for very large frequencies. This
is similar to our observations from the scope measurements, only easier to see.

But there is more! Enable [Cursors On] by checking the box.

Now drag the cursor line (red) on the left side until the phase is close to 45 degrees. This is a special point and is called the -3 dB cutoff frequency on the gain plot.

Recall the definition of the phase

\[ \tan \phi = \left( \frac{|X_c|}{|R|} \right) = \left( \frac{1}{\omega CR} \right) \]

At the angular frequency \( \omega \) where \( \phi = 45 \) degrees, \( \tan 45 = 1 \).

At this point, the real part of the impedance equals the imaginary part of the impedance.

In the real world, this means that the voltage across the capacitor \( V_C \) is equal to the voltage across the resistor \( V_R \). You can observe this from your Scope or Bode measurements.
Exercise 2-3: Where is the Capacitance/Inductance Meter?

NI myDAQ software for the DMM function does not support capacitance or inductance measurements. It is a bit of a long story, but more circuitry and more outputs would be required. The added footprint and redesign would be costly. So what’s a guy to do?

We already have the answer.

Find the frequency ($f$) where the magnitude of the real and imaginary parts are equal either from time measurements (Scope) or from the phase graph (Bode).

Calculate the angular frequency from $\omega = 2\pi f$.

Next, use the DMM(W) to measure the resistance $R$.

The capacitance is given by $C = 1/\omega R$.

Example:

$R = 990 \, \text{W}$, $f = 1.545 \, \text{kHz}$, $\omega = 9.7 \, \text{radians}$, and $C = 0.1 \, \text{mF}$.

Wow, that’s neat! Works just as well with inductors using $L = R/\omega$. 