How Uncertain are Your Measurements?

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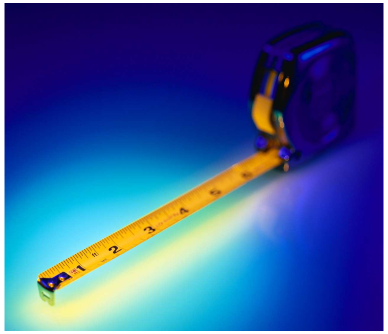
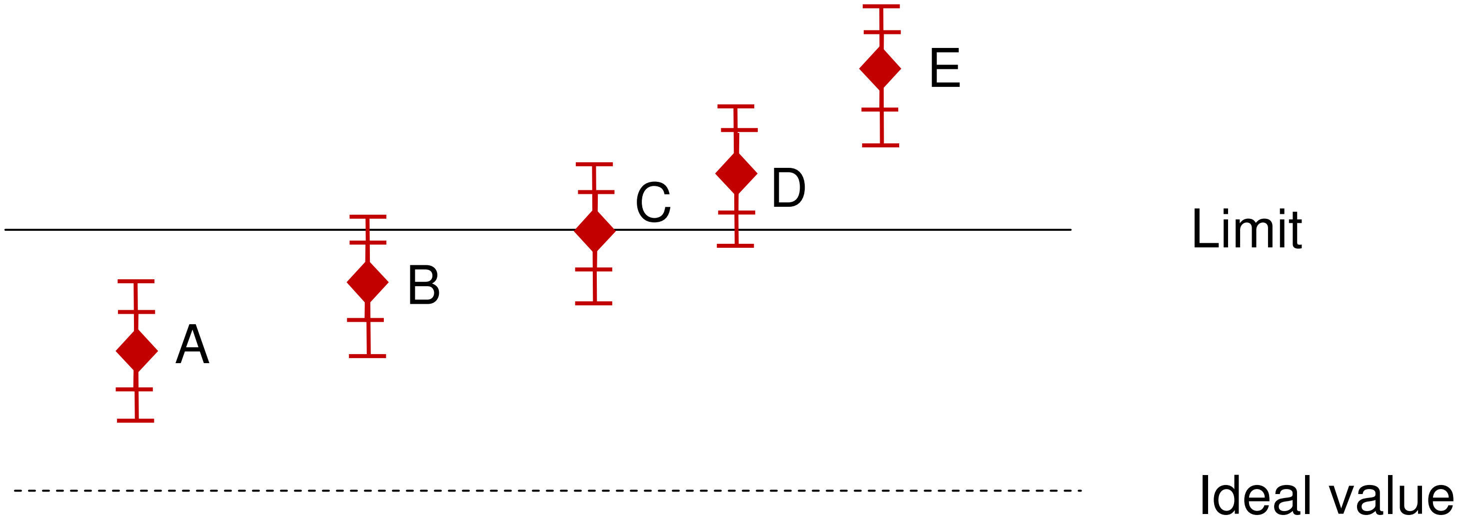
Measurement uncertainty

- A measurement result is an estimate of a quantity



- The estimate has an uncertainty
    - The measurement uncertainty can be large or small, but it is always present
- How to estimate the measurement uncertainty is described by *GUM* (*Guide to the Expression of Uncertainty in Measurement)*

Motivation



Distribution? Confidence level?

Why bother with uncertainty?

- What if our measurement are wrong?
  - Prolonged development time
  - Wrong decisions
  - Bad product quality
  - Products with good quality are unnecessarily discarded
  - Bad reputation among the customers
  - …

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Measurement uncertainty vs error

- **Error** – difference between measured value and the true value
- **Measurement uncertainty** – quantification of the uncertainty in the measured value
- The error is in most cases always unknown, as we have no knowledge about the true value. But we can estimate the measurement uncertainty.
- Errors which effect and size are known can be compensated for through adjustments

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Sources of measurement uncertainty

- Instrument
  - EMC, noise, age, wear and tear
- Object
  - The measured object may be instable (ex. measure the size of an ice cube in a warm room)
- Process
  - The measurement in itself may be difficult to perform
- Imported uncertainties
  - The calibration of an instrument has an uncertainty of its own
- User skill
  - Reading errors
- Data collection – the measured value has to be representative
  - Ex. do not always perform the measurement at the same time, make sure that the probe is correctly placed
- Environment
  - Temperature, humidity, air pressure, etc…

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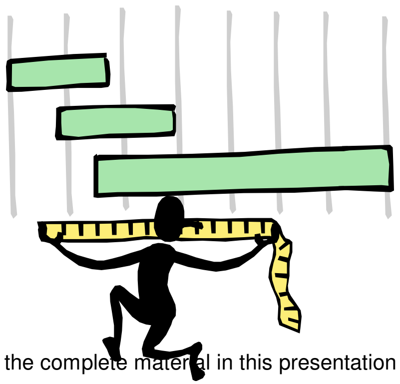
Two categories of standard uncertainty

- The uncertainty of a measurement result is composed of several components which may be evaluated in either of two ways:
  - Type A
    - Component which numerical value can be evaluated with statistical methods (for example through statistical analysis of an observation series)
  - Type B
    - Component which numerical value is evaluated by other methods (calibration certificate, manufacturer specification, computations, experience, common sense etc...)



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Probability distributions

- Gaussian distribution
- Uniform (rectangular) distribution
- U-shaped distribution

Type A standard uncertainty

- An estimate q of the quantity Q is provided from *n* statistically independent observations q_i by computing the arithmetic mean:

$$\bar{q} = \frac{1}{n}\sum_{i=1}^{n} q_i$$

- The standard uncertainty is the standard deviation of the mean value and is calculated as:

$$u(\bar{q}) = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}\left(q_i - \bar{q}\right)^2}$$

Note! Keep n ≥ 10

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Type B standard uncertainty

- Single value exists
  - Use the value
  - Use given uncertainty value or estimate it from experience

- Known higher and lower limits
  - Assume a probability distribution. Most often a rectangular distribution is used, but Gaussian and U-shaped distributions are also quite common.
  - Example: In the case of a rectangular probability distribution with max value a_+ and min value a_- the measurement uncertainty is given by:

$$u(x_i) = \sqrt{\frac{1}{12}(a_+ - a_-)^2}$$

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Combined standard uncertainty 1(2)

- If a quantity *Y* is calculated according to the algorithm *f* with several inputs *X*, then the standard uncertainty is calculated as follows:

  - Estimation of *Y*:

$$y = f\left(x_1, x_2, \ldots, x_N\right)$$

  - Combined measurement uncertainty $u_c(y)$:

$$u_c(y) = \sqrt{\sum_{i=1}^{N}\left(u(x_i)\frac{\partial f}{\partial x_i}\right)^2} \quad , \quad c_i = \frac{\partial f}{\partial x_i}$$

- The sensitivity factor c_i describes to which degree the estimation of *y* is influenced by the quantity x_i

Combined standard uncertainty 2(2)

- Example:
  - Addition and subtraction: y = a + b - c

$$u(y) = \sqrt{u(a)^2 + u(b)^2 u(c)^2}$$

  - Multiplication and division: y = ab/c

$$u(y) = y\sqrt{\frac{u(a)^2}{a^2} + \frac{u(b)^2}{b^2} + \frac{u(c)^2}{c^2}}$$

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Measurement uncertainty budget

- The budget provides a clear picture of the uncertainty sources, their contributions and the total measurement uncertainty.

Quantity	Estimate	Standard deviation	Probability distribution	Sensitivity factor	Contribution to standard uncertainty
X_1	x_1	$u(x_1)$		c_1	$u(y_1)$
X_2	x_2	$u(x_1)$		c_2	$u(y_2)$
:	:	:		:	:
X_N	x_N	$u(x_N)$		c_N	$u(y_N)$
Y	y				$u(y)$

Expanded uncertainty

- The coverage factor *k* is used to scale a combined standard uncertainty to another confidence level
- The expanded uncertainty (U)

$$U = ku(y) \quad , \ k \text{ is the coverage factor}$$

- Example of coverage factors for the Gaussian distribution:
  - k = 1, confidence level 68 %
  - k = 2, confidence level 95 %
  - k = 2,58, confidence level 99 %
  - k = 3, confidence level 99,7 %

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Step by step

1. Describe what you want to measure, for example make a model that shows the relation between the measured quantity and other quantities and express this in an equation
2. Identify the largest sources to the measurement uncertainty
3. Evaluate the size of the contributions from these sources and express them in a uniform way using standard uncertainties
4. Combine these contributions into a combined standard uncertainty
5. Scale the result with the chosen coverage factor

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Measurement uncertainty for virtual instruments 1(2)

- Natural instrument
  - Specifications and calibration certificates are provided by the manufacturer

- Virtual instrument
  - The developer must evaluate the measurement uncertainty of the virtual instrument and if needed calibrate it
    - This may be a difficult task
  - One solution is to by virtual instruments for which the measurement uncertainty is specified
    - Rare solution

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Measurement uncertainty for virtual instruments 2(2)

- Evaluate the combined standard uncertainty
  - Calibration
    - Expensive and it only applies for the used test objects and conditions
  - Analytical methods
    - Feasible for simple algorithms
    - Awkward or impossible for more complicated algorithms, this particularly applies to non-linear functions
  - Numerical methods
    - Monte-Carlo simulation

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Monte Carlo simulation

- A method for *iteratively* evaluating a deterministic model using sets of random numbers as inputs
- MC simulation is often used when the model is complex, nonlinear, or involves more than just a couple of uncertainty sources
- Advantages
  - The full probability distribution is obtained
  - Straightforward approach (even for complicated algorithms)
  - Makes it possible to compute the measurement uncertainty on the fly for a specific set of measured values

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Step by step

Numerical methods

1. State a suitable probability distribution for each uncertainty contribution. Each of which is evaluated using type A or B approaches
2. Perform a large number of simulations (> 10000). Input data is sampled from the from the distributions described in step 1.
3. The output result is a probability distribution, the standard deviation of which is used as the measurement uncertainty.

Important: Use a good random number generator

Questions?

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