Høgskolen i Telemark

Presentation at NI Day 2010
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Lillestrøm, Norway
(http://www.ni.com/norway/nidays)

MPC vs. PID

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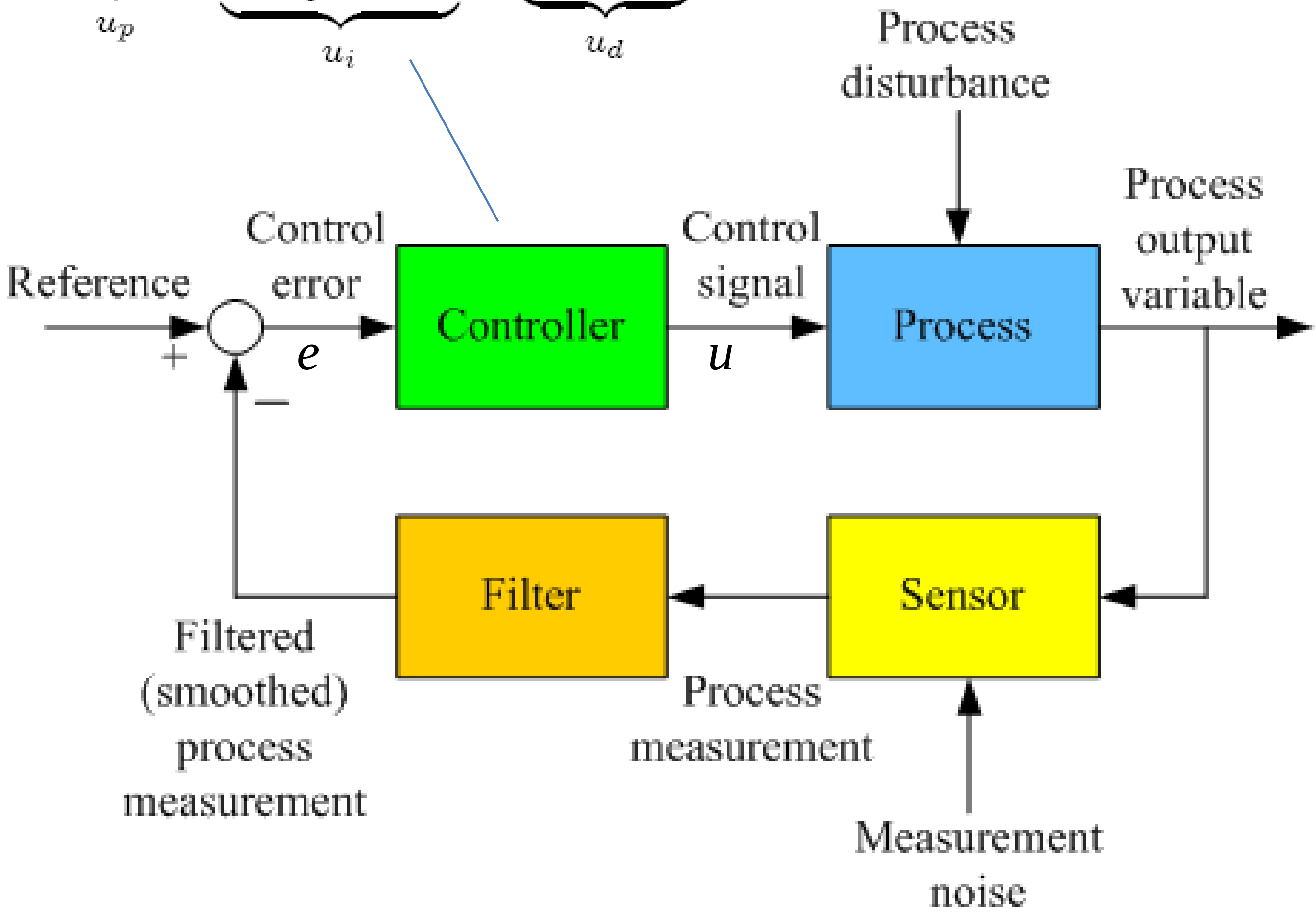
Agenda:

- PID control (Proportional + Integral + Derivative)
- MPC (Model-based Predictive Control)
- System used in practical demo: Air heater (temperature control)
- Adapting mathematical model to physical system
- PID settings based on Skogestad's model-based tuning
- MPC settings
- PID vs. MPC:
  - Setpoint tracking
  - Disturbance compensation (non-modeled disturbance)
  - Propagation of measurement noise through the controller
  - Robustness of control system stability against model error
  - Control when the process output variable is constrained
- More about MPC:
  - Playing with prediction and control horizons
  - Playing with weight of control signal increment
- Conclusions

PID control

$$u = u_0 + \underbrace{K_p e}_{u_p} + \underbrace{\frac{K_p}{T_i}\int_0^t e \, d\tau}_{u_i} + \underbrace{K_p T_d \frac{de}{dt}}_{u_d}$$



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Model-based Predictive Control (MPC)

The Control Design and Simulation module of LabVIEW contains an MPC controller

Process model:

$$x(k+1) = Ax(k) + Bu(k)$$

$$y(k) = Cx(k) + Du(k)$$

Constraints:

$$y_{\min} \le y \le y_{\max}$$

$$\Delta u_{\min} \le \Delta u \le \Delta u_{\max}$$

$$u_{\min} \le u \le u_{\max}$$

Optimization criterion:

$$= \sum_{k=0}^{N_p} \left\{ Q_1 \left[e_1(t_k)\right]^2 + Q_2 \left[e_2(t_k)\right]^2 + \ldots + Q_n \left[e_n(t_k)\right]^2 \right\}$$

$$+ \sum_{k=1}^{N_c} R_1 \left\{ \left[\Delta u_1(t_k)\right]^2 + R_2 \left[\Delta u_2(t_k)\right]^2 + \ldots + R_r \left[\Delta u_r(t_k)\right]^2 \right\}$$

$$= \text{Sum of future weighed squared control errors}$$

$$+\text{Sum of future weighed increments of control variable}$$

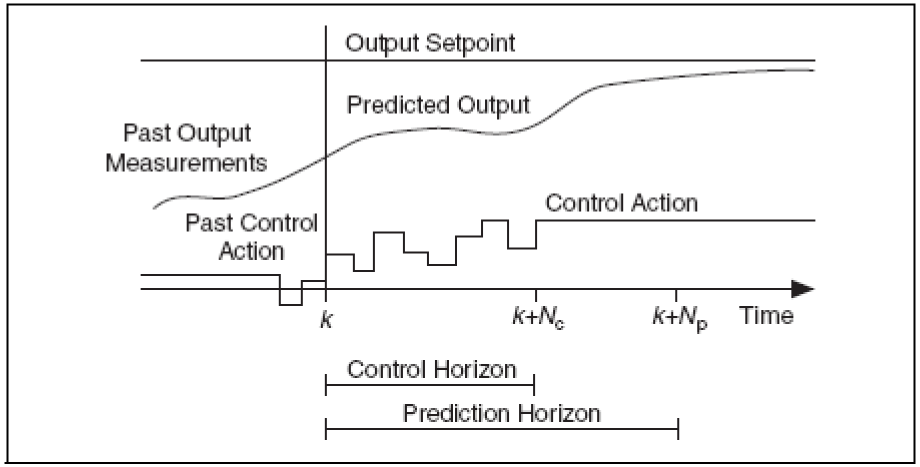

Figure 18-1. Prediction and Control Horizons

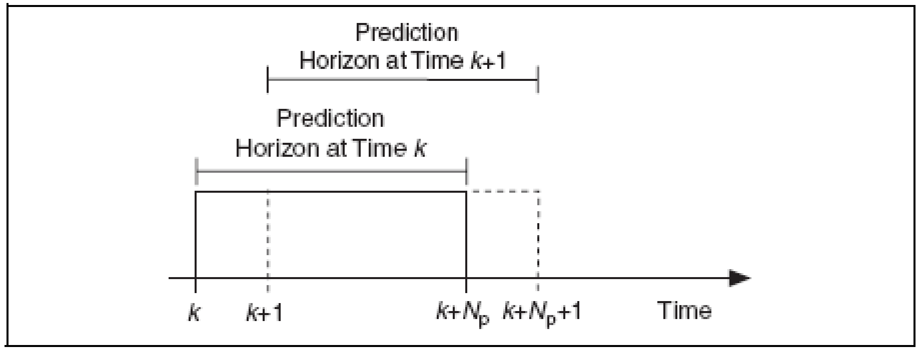

Figure 18-2. Moving the Prediction Horizon Forward in Time

(Figures from user manual of Control Design and Simulation module)

MPC

Assumed process model (discrete-time linear state-space model):

$$x(k+1) = Ax(k) + Bu(k)$$

$$y(k) = Cx(k) + Du(k)$$

System matrices A, B, C, D must have known values.
Can be found from physical laws or from system identification based on experiments.

MPC

Constraints to be defined (these are a part of the MPC controller):

$$y_{\min} \le y \le y_{\max}$$

$$\Delta u_{\min} \le \Delta u \le \Delta u_{\max}$$

$$u_{\min} \le u \le u_{\max}$$

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MPC

Optimization criterion

(Will be minimized by the controller,
typically using a QP algorithm (Quadratic Programming))

$$
\begin{aligned}
&= \sum_{k=0}^{N_p} \left\{ Q_1 \left[e_1(t_k)\right]^2 + Q_2 \left[e_2(t_k)\right]^2 + \ldots + Q_n \left[e_n(t_k)\right]^2 \right\} \\
&\quad + \sum_{k=1}^{N_c} R_1 \left\{ \left[\Delta u_1(t_k)\right]^2 + R_2 \left[\Delta u_2(t_k)\right]^2 + \ldots + R_r \left[\Delta u_r(t_k)\right]^2 \right\} \\
&= \text{Sum of future weighed squared control errors} \\
&\quad + \text{Sum of future weighed increments of control variable}
\end{aligned}
$$

MPC

How it works:

Present control signal is calculated from optimal future control system behaviour:

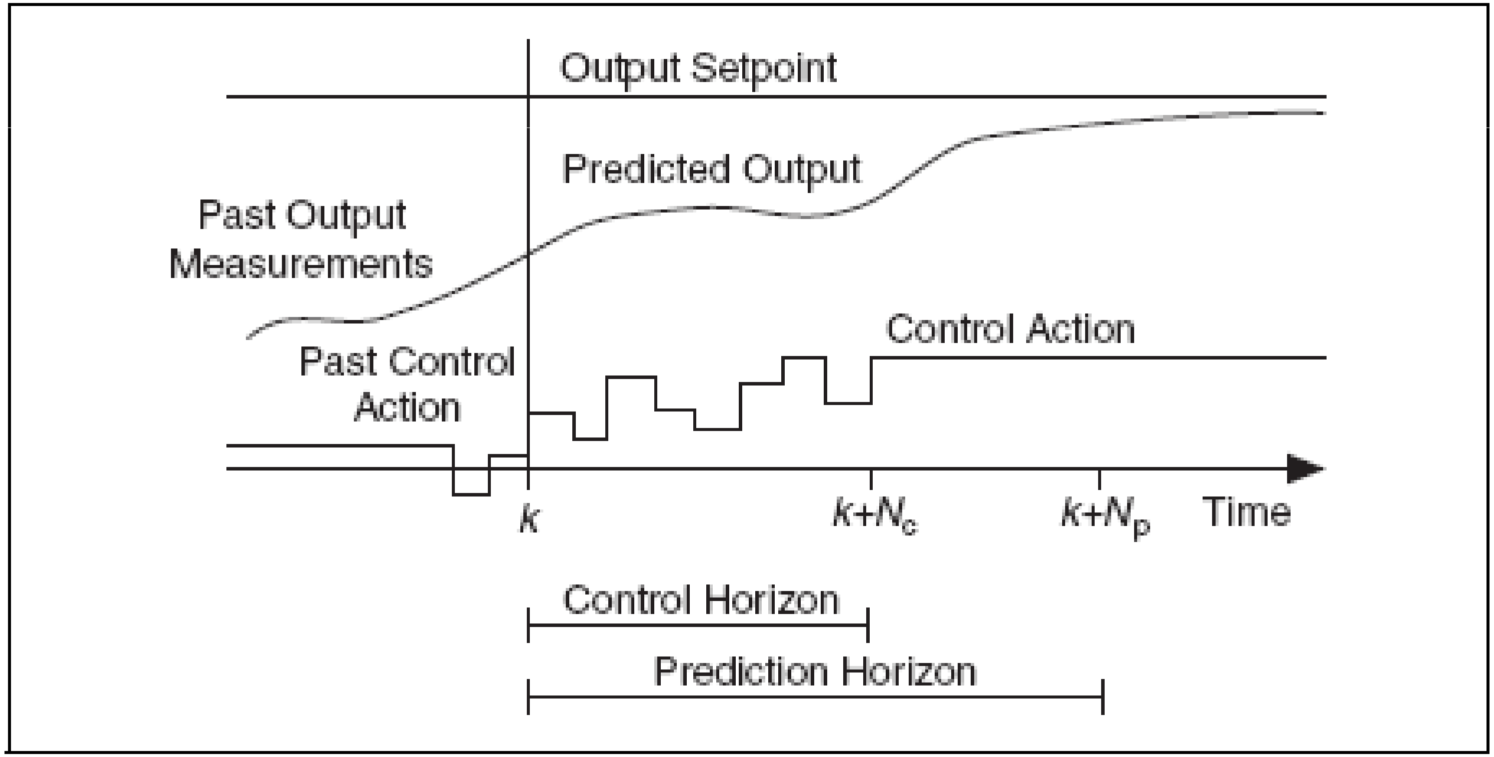
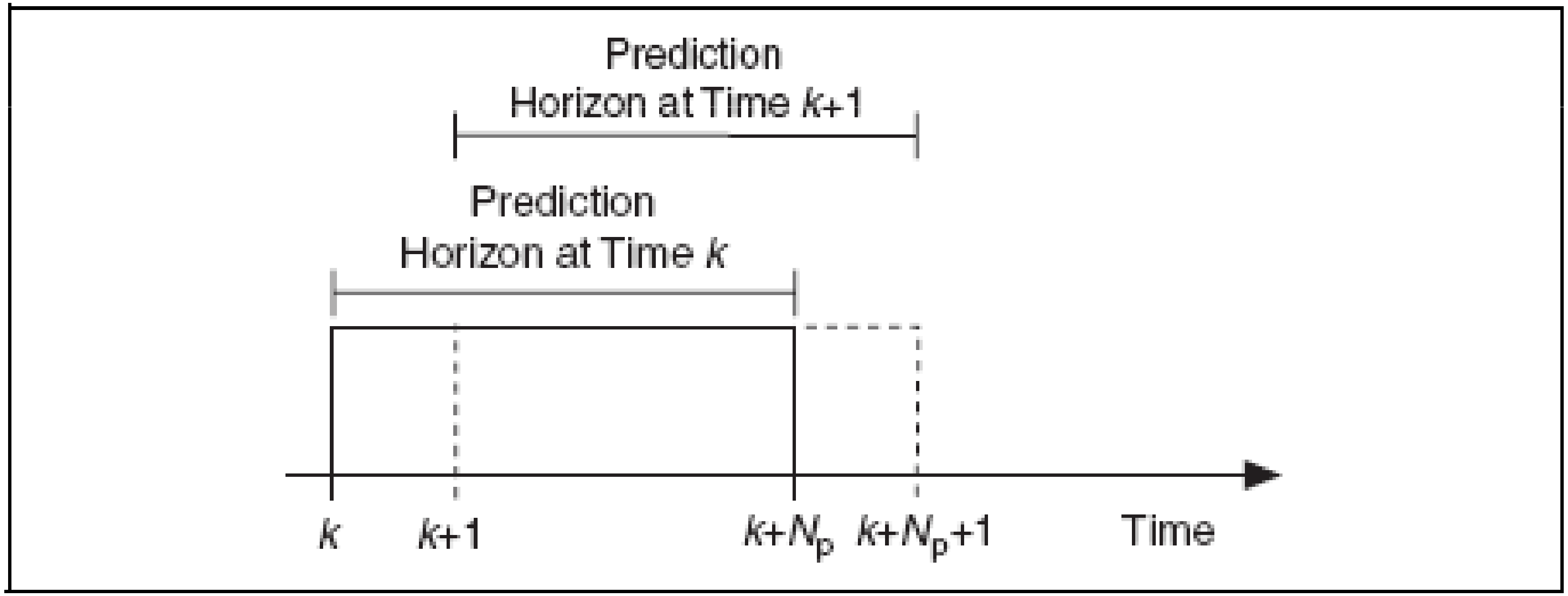


Figure 18-1. Prediction and Control Horizons

MPC

How it works:

Prediction horizon is moved ahead as time goes (the moving horizon principle):



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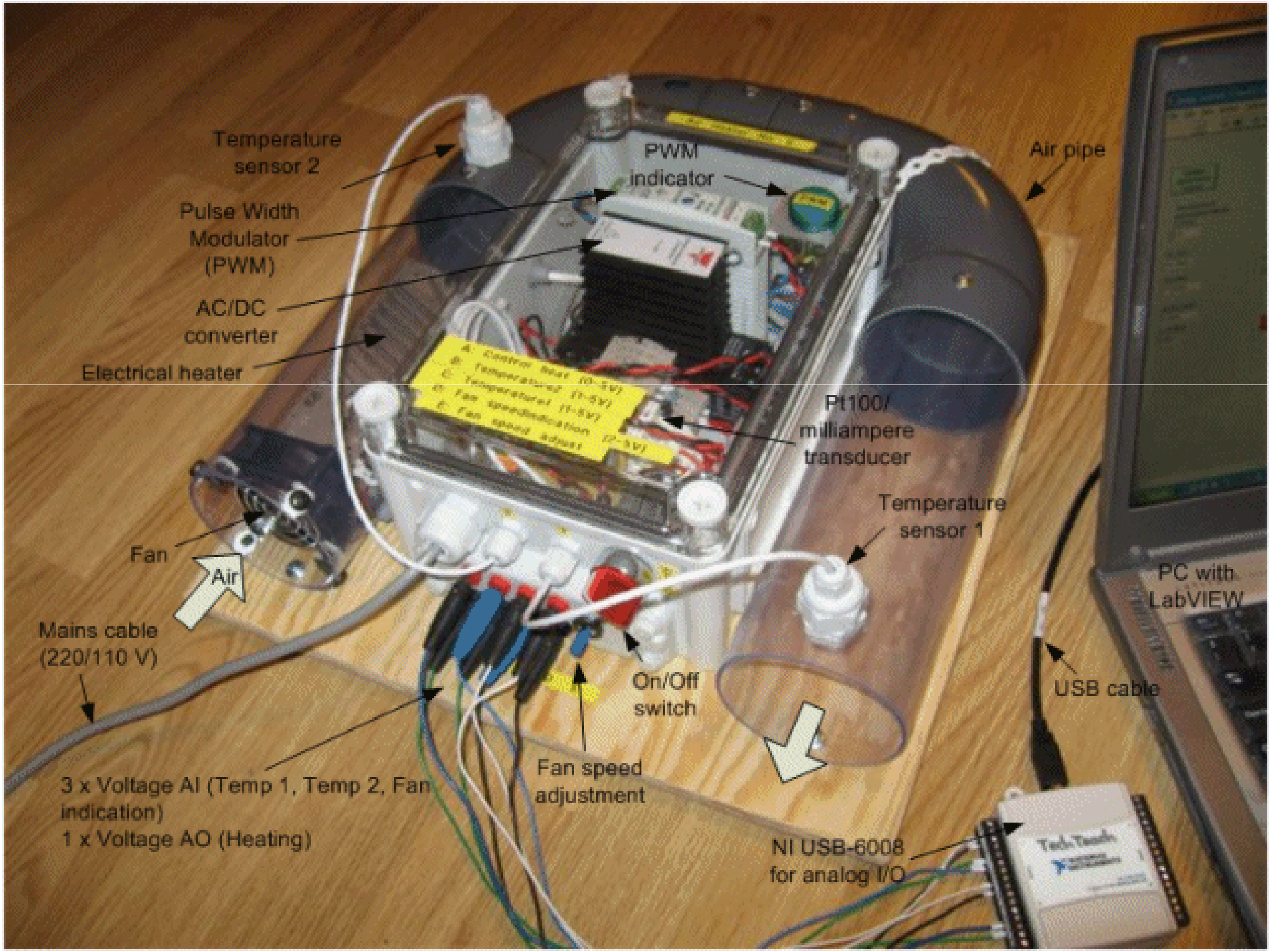
Air heater

Temperature at outlet to be controlled by adjusting heat



More info about the air heater at http://home.hit.no/~finnh/air_heater

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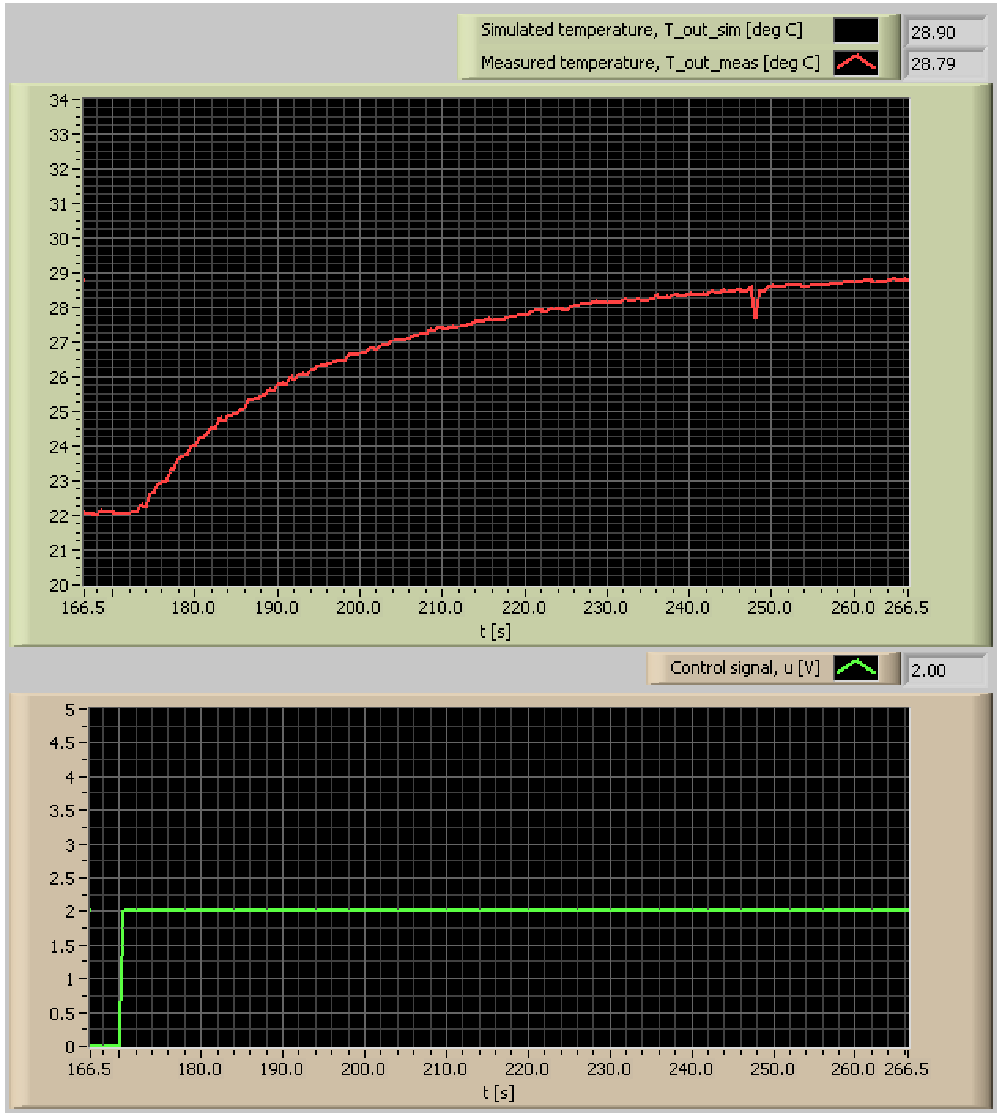
Adapting the model to physical system

Step response of process:



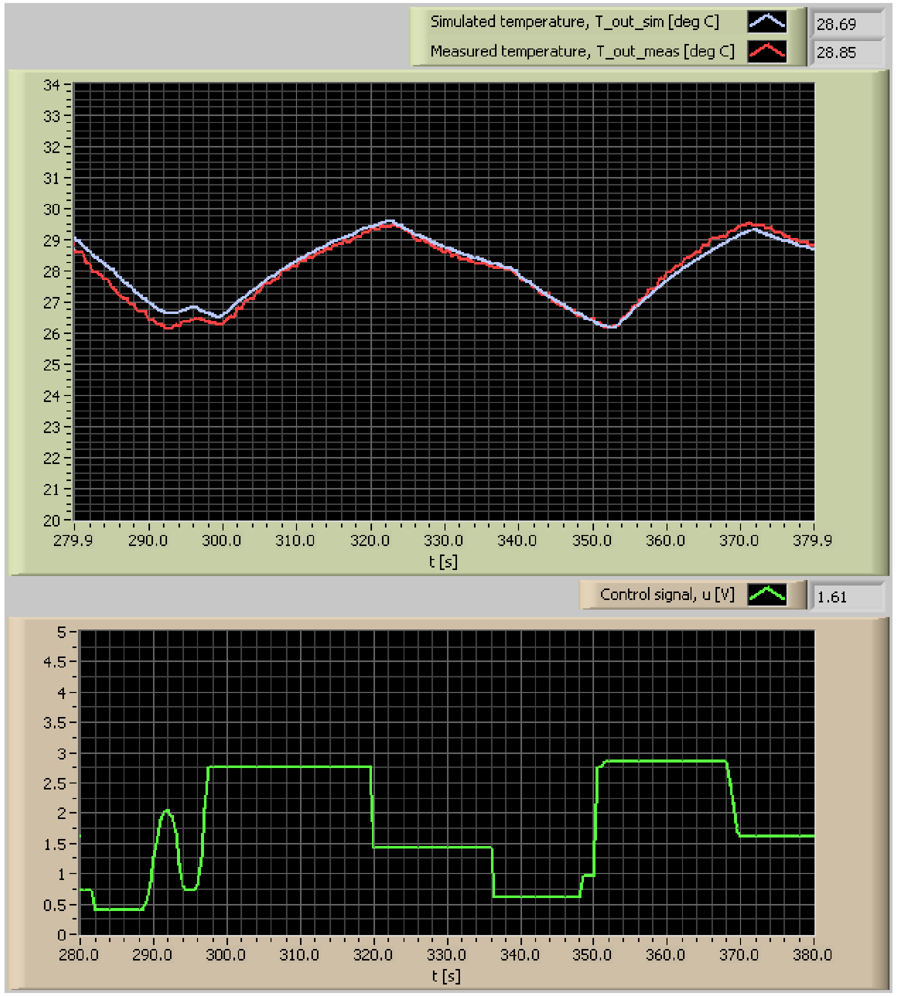
Step in control signal:

Gain = 3.5

Time constant = 22 sec

Time delay = 2 sec

Verifying the model, and possibly fine-tuning the model parameters, by running a simulator in parallel with the physical system:



Model seems to be excellent!

Many methods for system identification exist, and are implemented in System Identification Toolkit, but the simple approach described above works fine here.

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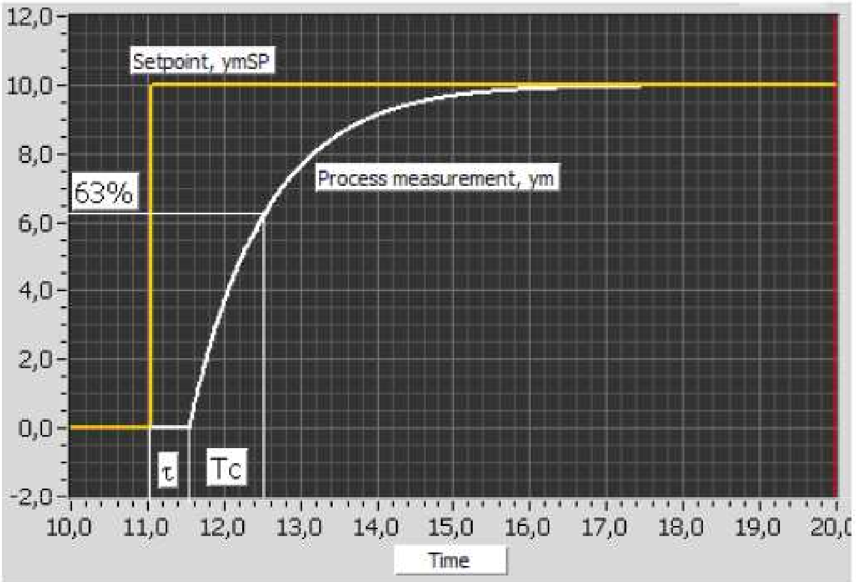
Skogestad's PID tuning method

Specified closed-loop step resonse in terms of time-constant Tc:



PI(D) tuning formulas for various process models

Our model: ⟹ (Time-constant + delay)

Process type	$H_{psf}(s)$ (process)	K_p	T_i	T_d
Integrator + delay	$\frac{K}{s}e^{-\tau s}$	$\frac{1}{K(T_C+\tau)}$	$c\,(T_C+\tau)$	0
Time-constant + delay	$\frac{K}{Ts+1}e^{-\tau s}$	$\frac{T}{K(T_C+\tau)}$	$\min\left[T,\ c\,(T_C+\tau)\right]$	0
Integr + time-const + del.	$\frac{K}{(Ts+1)s}e^{-\tau s}$	$\frac{1}{K(T_C+\tau)}$	$c\,(T_C+\tau)$	T
Two time-const + delay	$\frac{K}{(T_1s+1)(T_2s+1)}e^{-\tau s}$	$\frac{T_1}{K(T_C+\tau)}$	$\min\left[T_1,\ c\,(T_C+\tau)\right]$	T_2
Double integrator + delay	$\frac{K}{s^2}e^{-\tau s}$	$\frac{1}{4K(T_C+\tau)^2}$	$4\,(T_C+\tau)$	$4\,(T_C+\tau)$

Skogestad's PID tuning method cont.

For the air heater:

$K = 3.5$

$T = 22$ s

$T_{delay} = 2$ s

Specification (a little arbitrary):

$T_C = 10$ s

PI tuning with Skogestad (time-constant with time-delay):

$K_p = T/(K*(T_C+T_{delay})) = 22/(3.5*(10+2)) = 0.42$

$T_i = \min(T, 1.5*(T_C+T_{delay})) = \min(22, 1.5*(10+2)) = 18$ s

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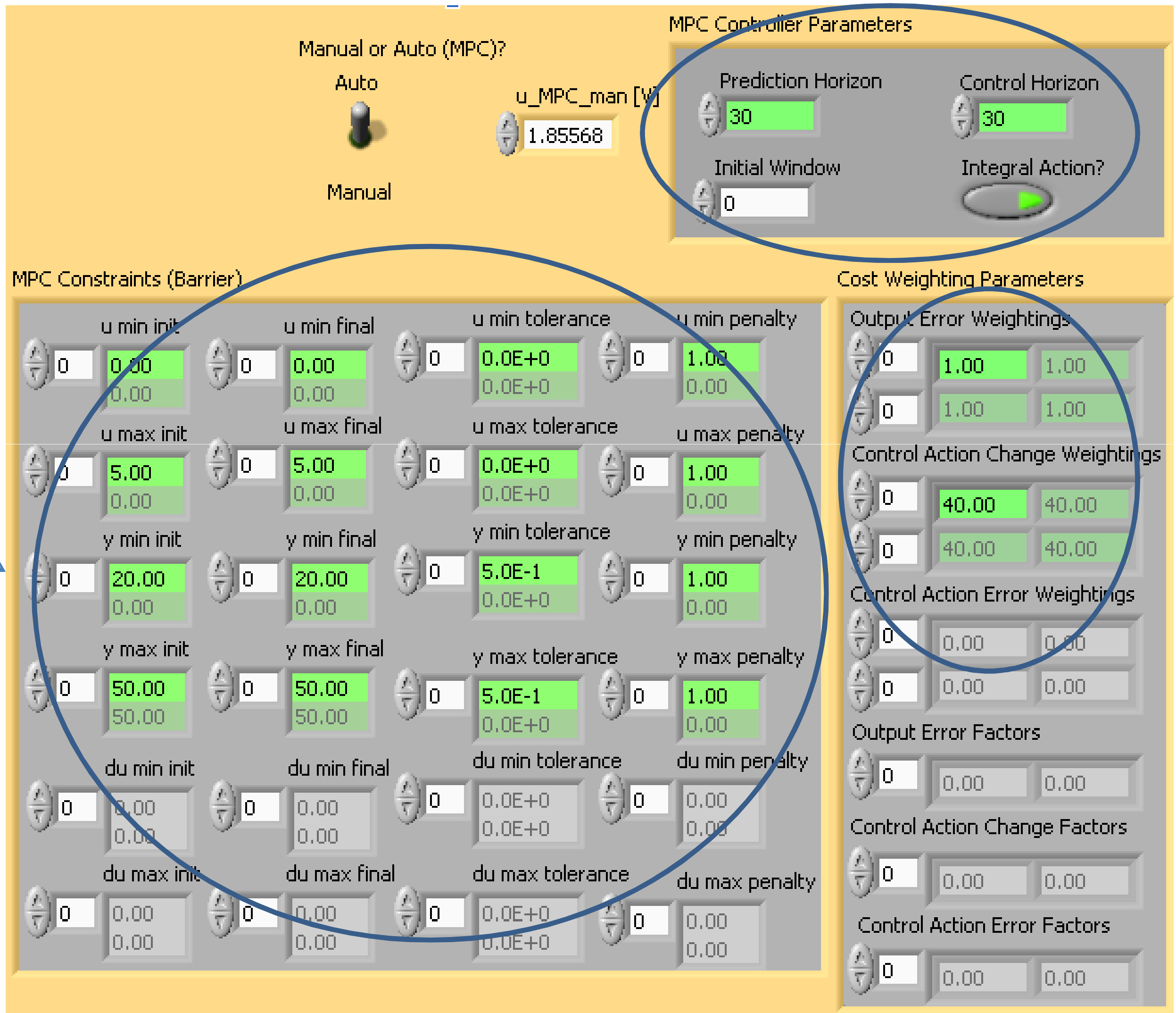
MPC settings



Set the constraints, typically according to physical limits

Set not so different from process response time (time-constant). 30 is number of samples. Sampling time is 0.5 s, hence horizon is 15 sec. Time-constant is 22 sec.

Output Error Weightings can be set to 1. Then adjust Control Action Change Weightings by trial-and-error on real system or simulator (small value gives fast, abrupt control; large gives sluggish control)

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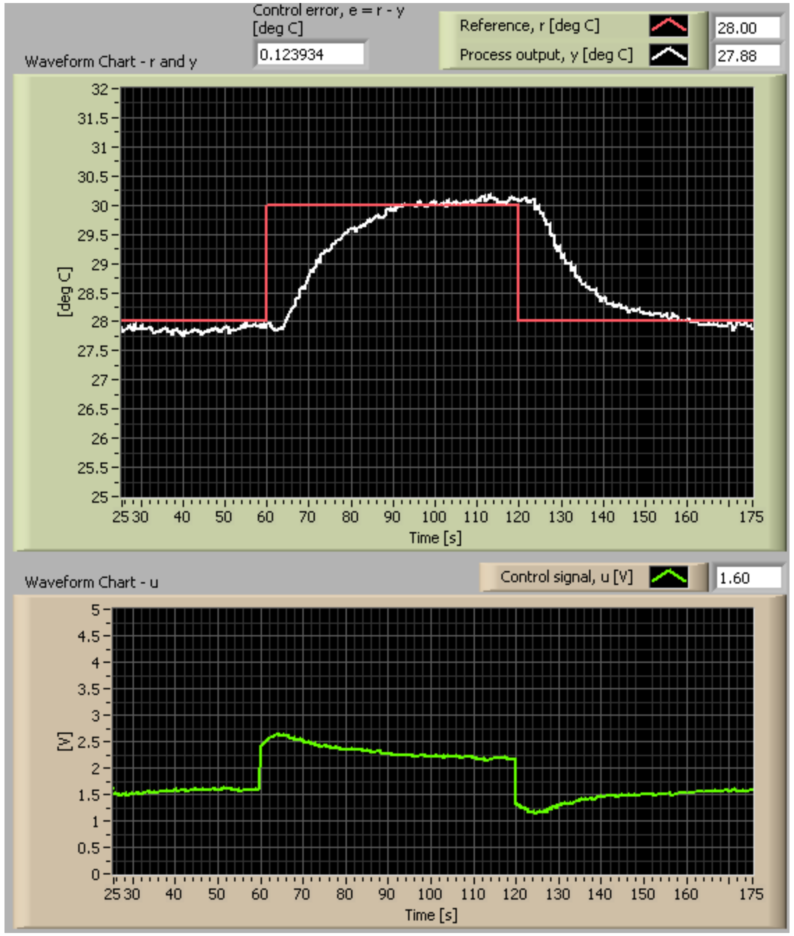
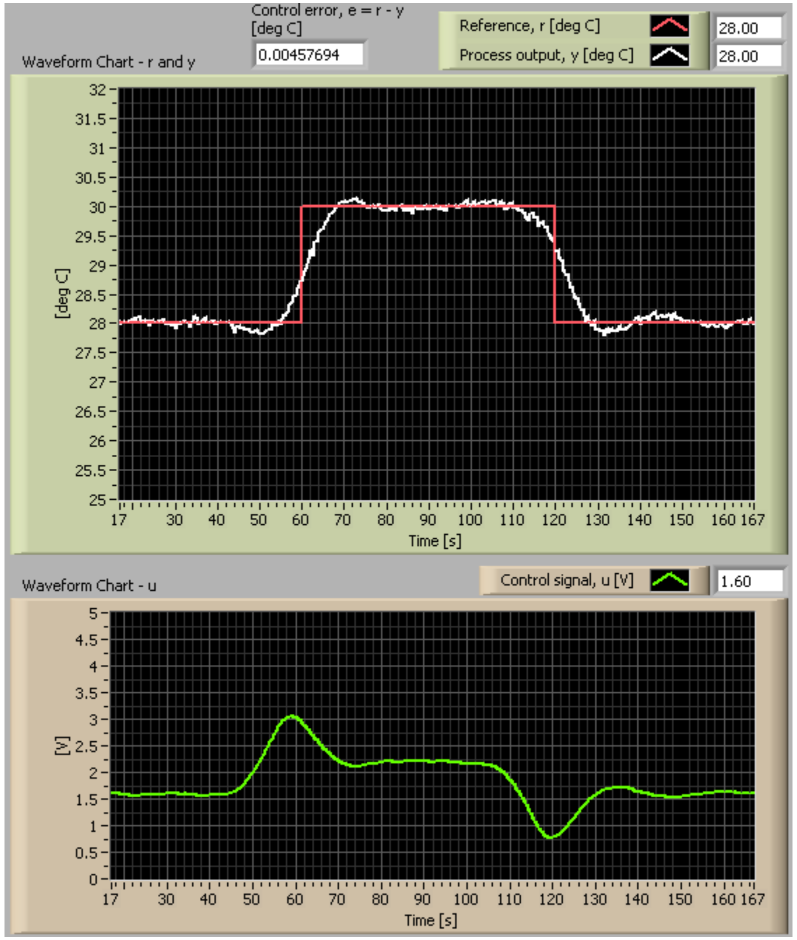
PID vs. MPC

Setpoint step tracking (future setpoint is known)

PID



MPC



MPC much better then PID, because MPC plans control by looking ahead.

Observe that MPC starts changing control ahead of setpoint change!

PID changes control after setpoint is changed.

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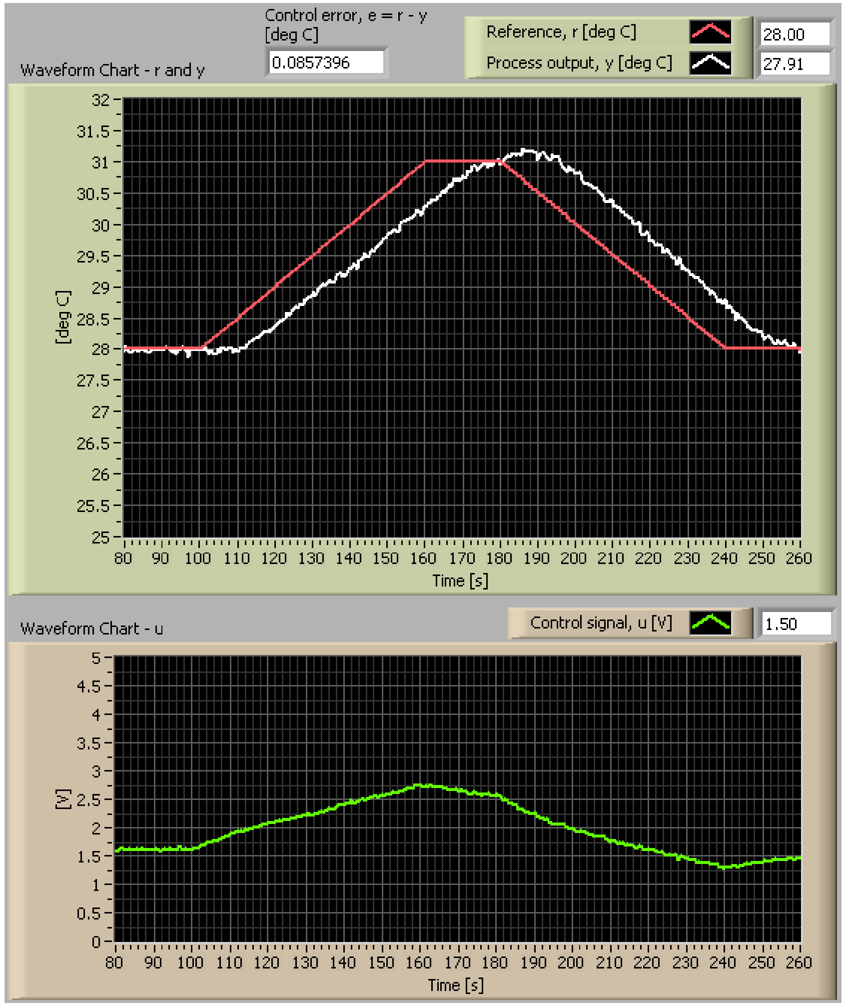
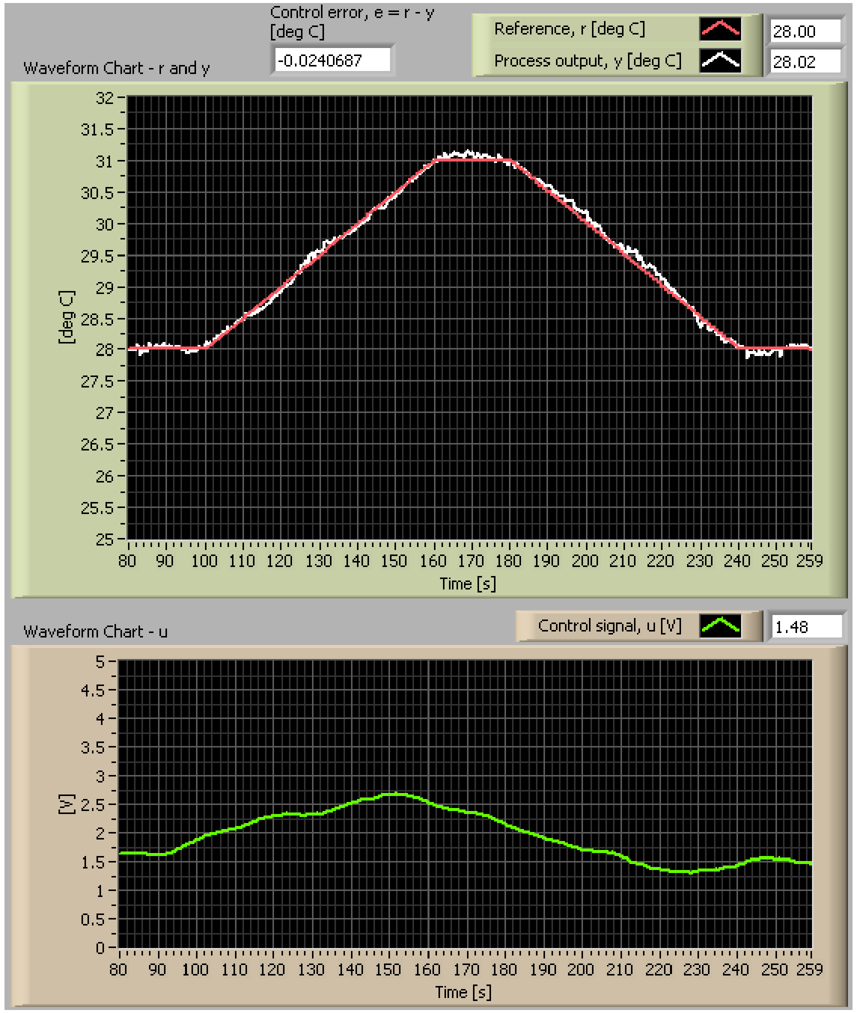
PID vs. MPC

Setpoint ramp tracking (future setpoint profile is known)

PID



MPC



☺

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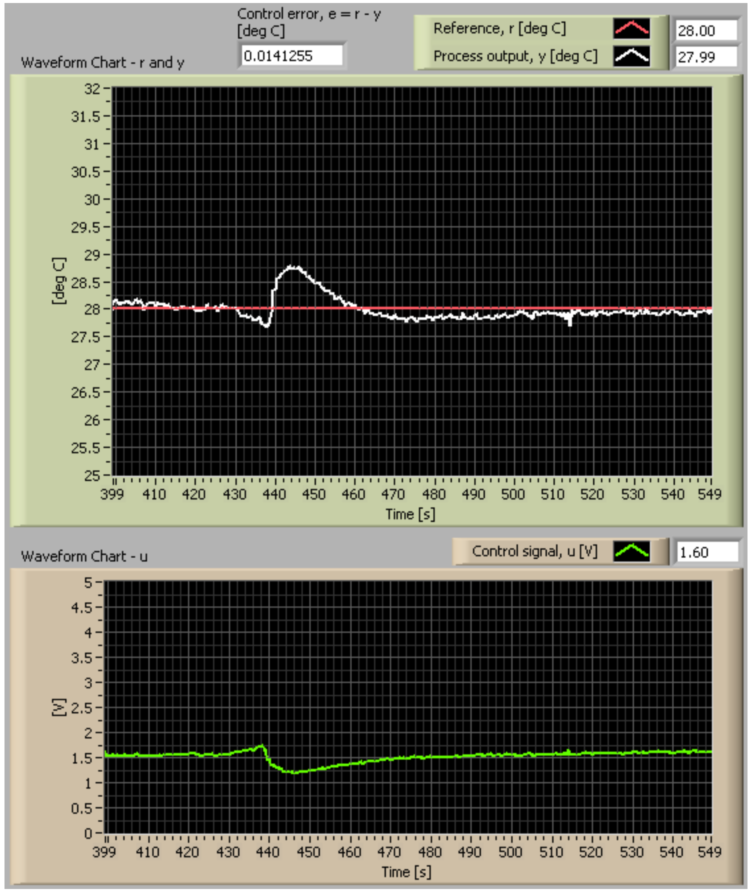
PID vs. MPC

Disturbance compensation

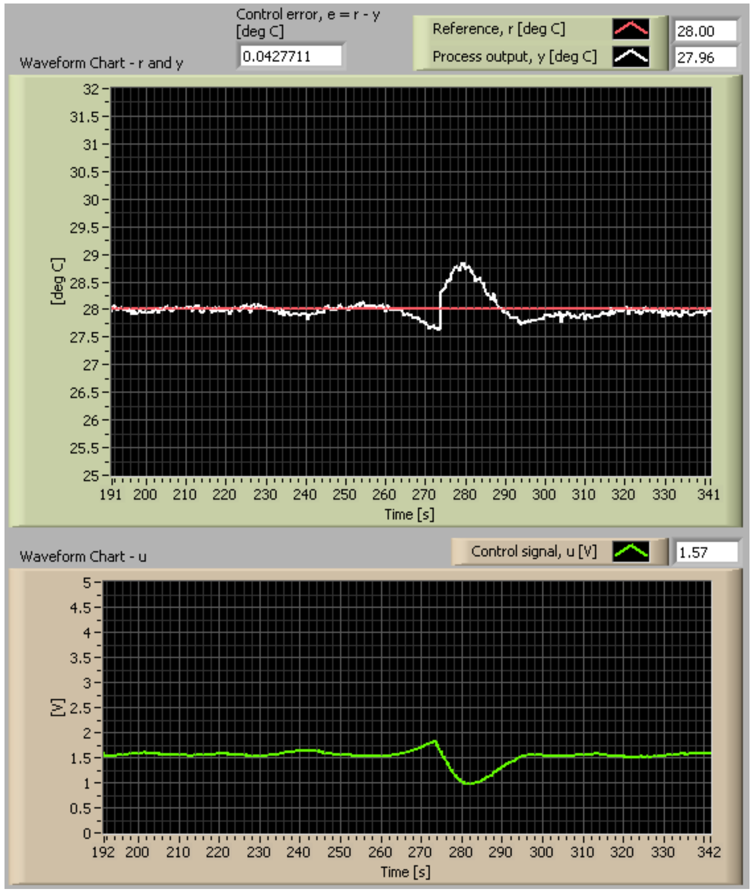
Disturbance = covering air inlet with hand for 10 sec.

(Disturbance is not known in advance, and not measured.)

PID



MPC



Not much difference between MPC and PID.

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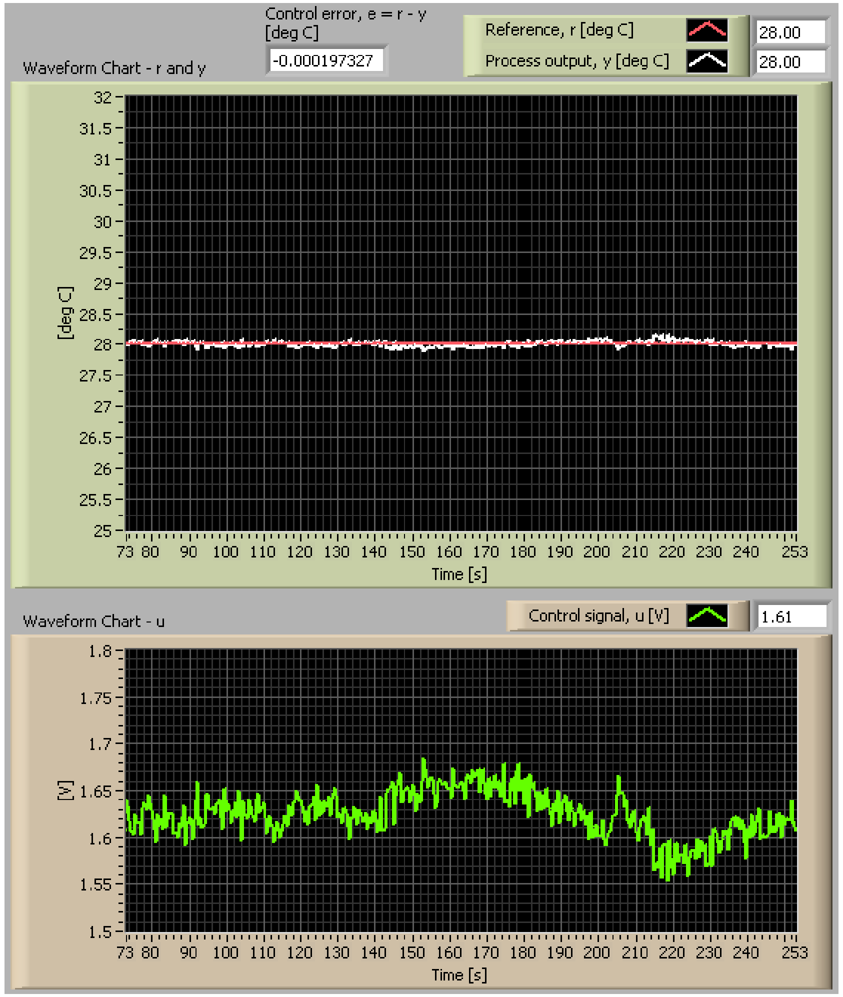
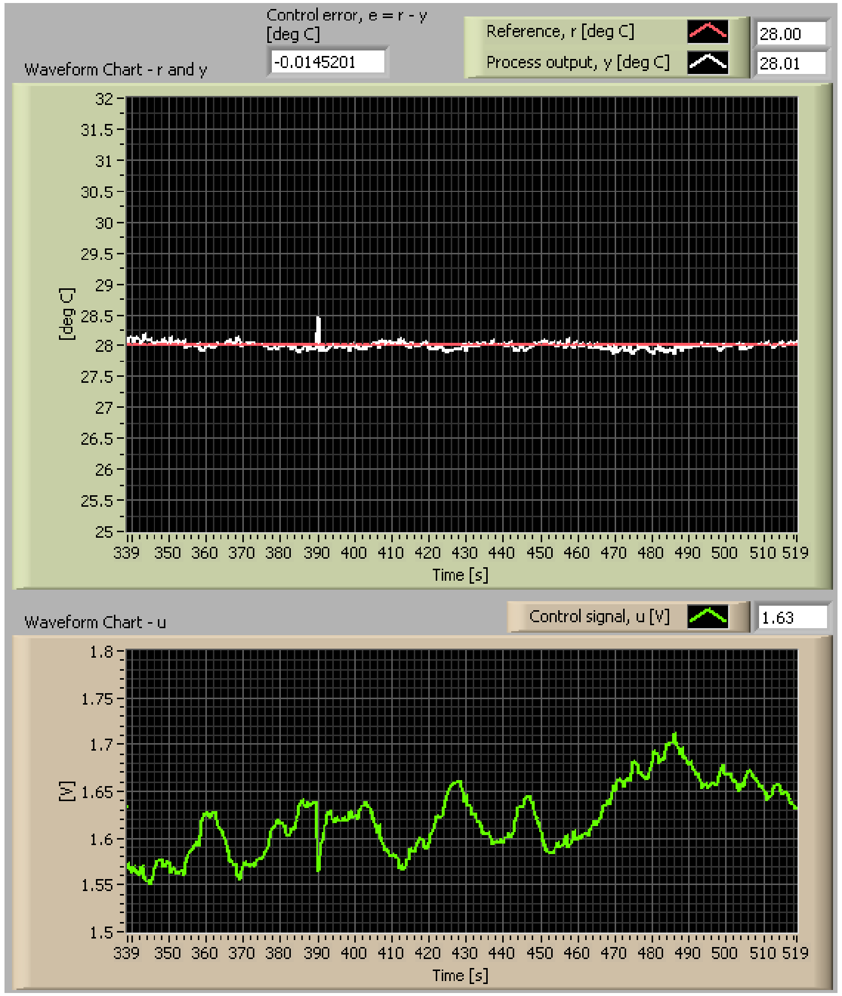
PID vs. MPC

Propagation of measurement noise through controller

PID



MPC



Smoother control signal with MPC (less propagation of noise through controller)

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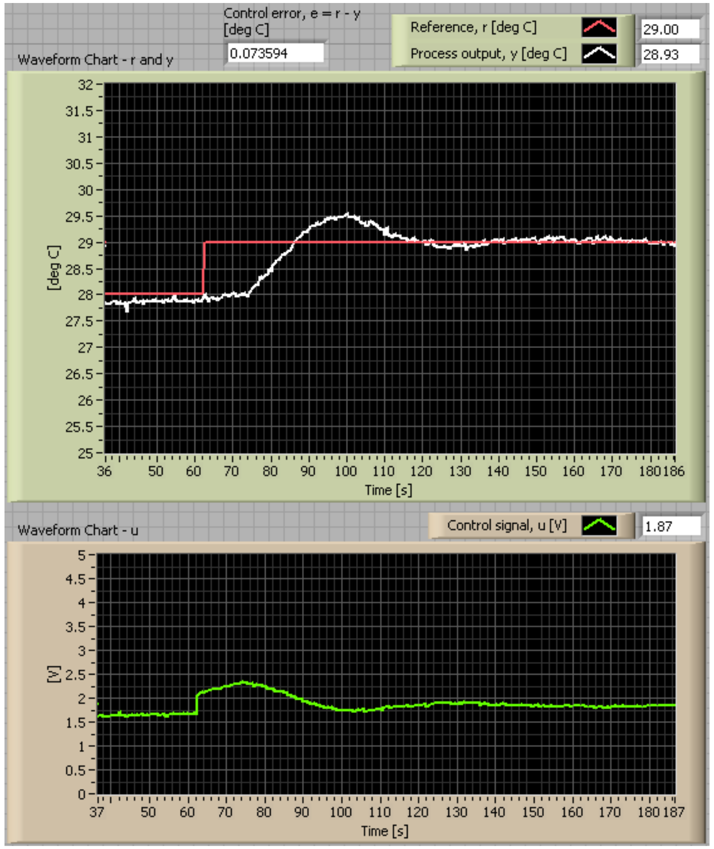
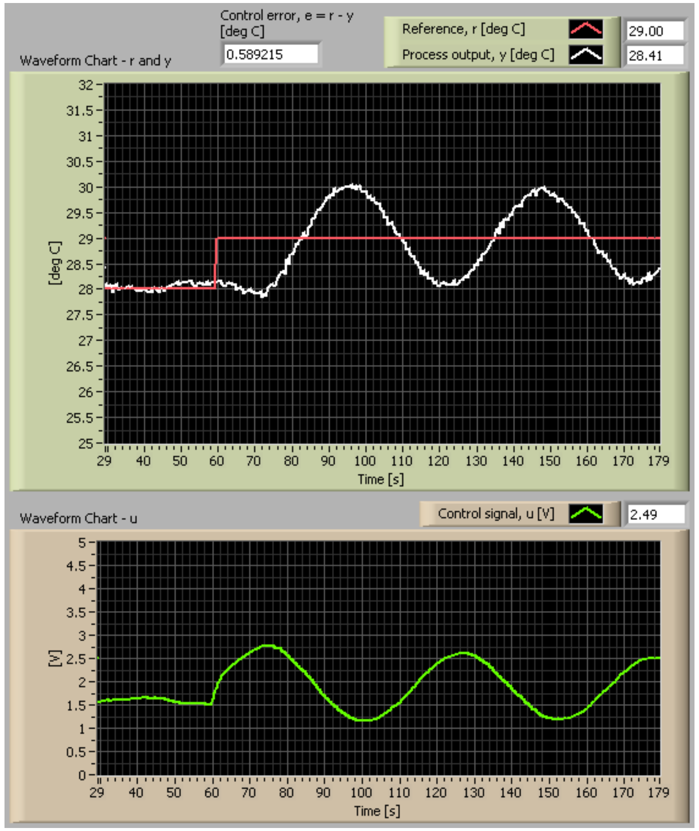
PID vs. MPC

Robustness against model change (error): Increase of loop time-delay by 8 sec

PID



MPC



MPC is less robust against this model change.
Makes sense because MPC is highly model-based.

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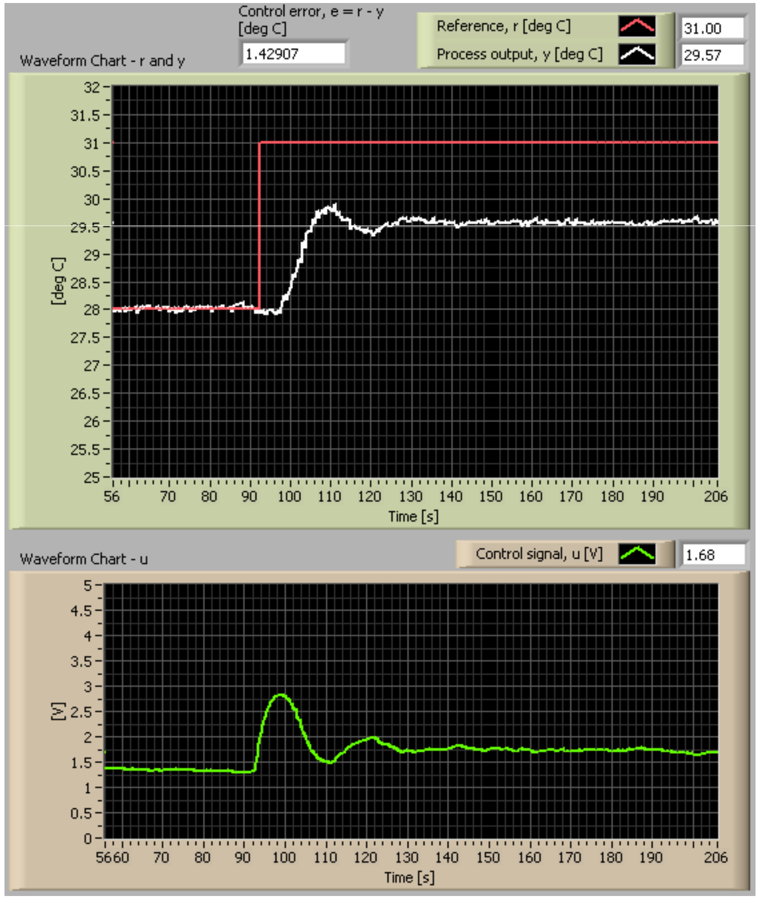
More about MPC: Constrained control

Control when output variable (temperature) is defined as constrained. Here: Max output (temperature) is set to 30 deg C, with a tolerance of 0.5 deg C (to avoid oscillations just below the limit).



Max limit of 30 deg C is maintained !

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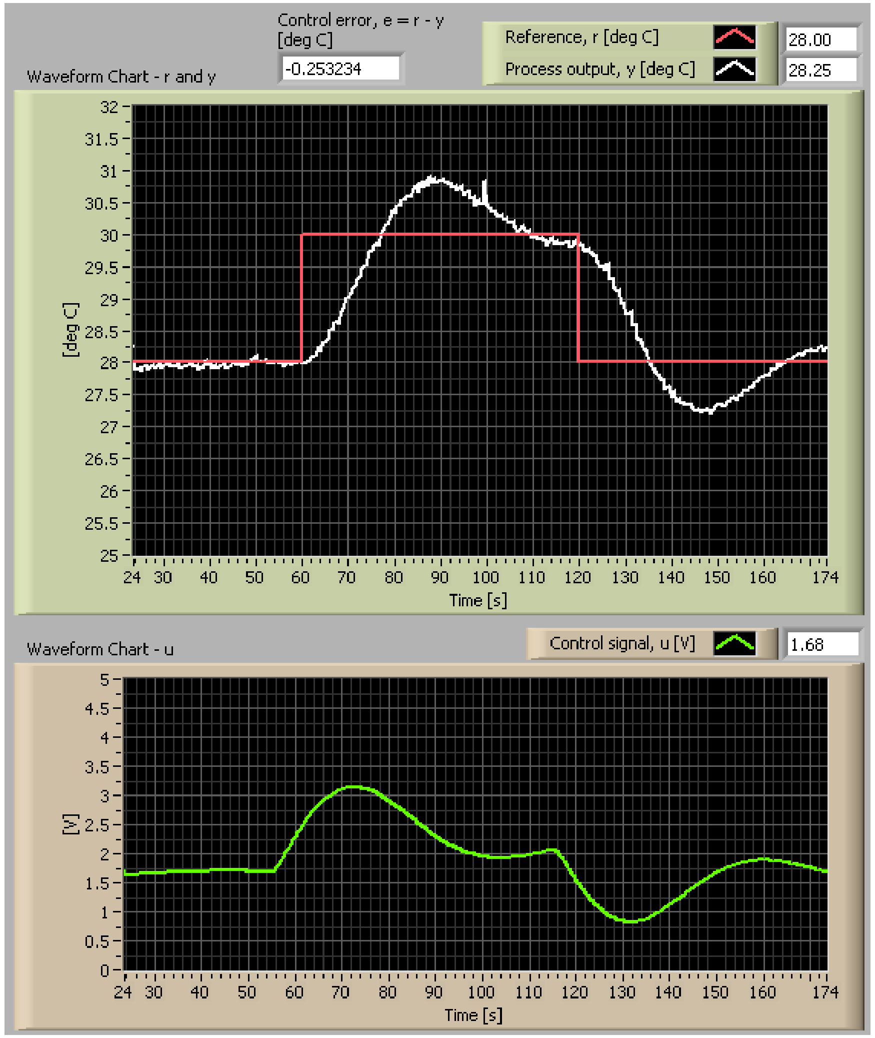
More about MPC: Decreasing control horizon

Prediction horizons reduced from 30 to 5 sec:



More sluggish and less stable control. May be explained by the controller taking only little (short-termed) future behaviour of control system into account when calculating control signal.

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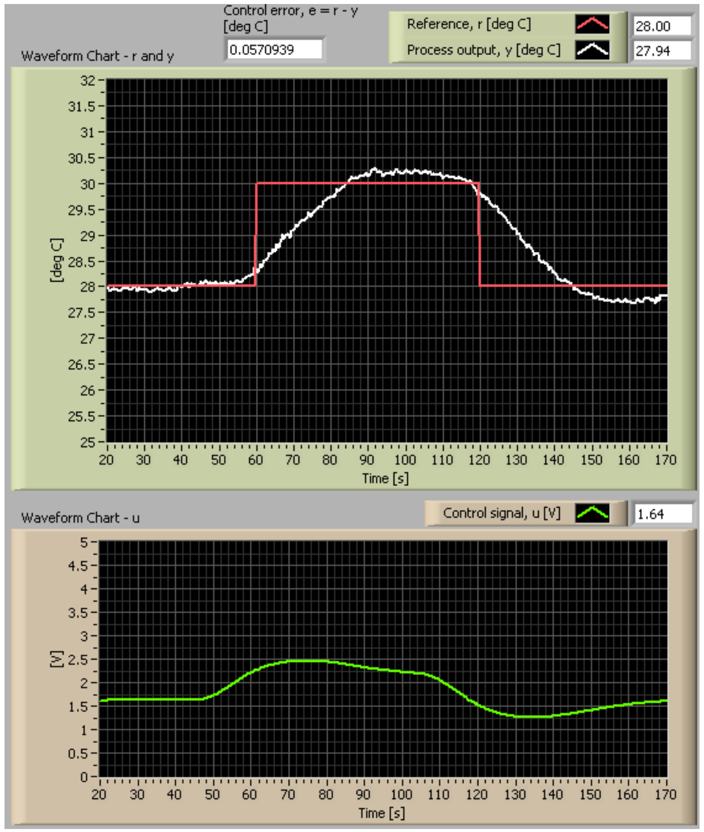
More about MPC

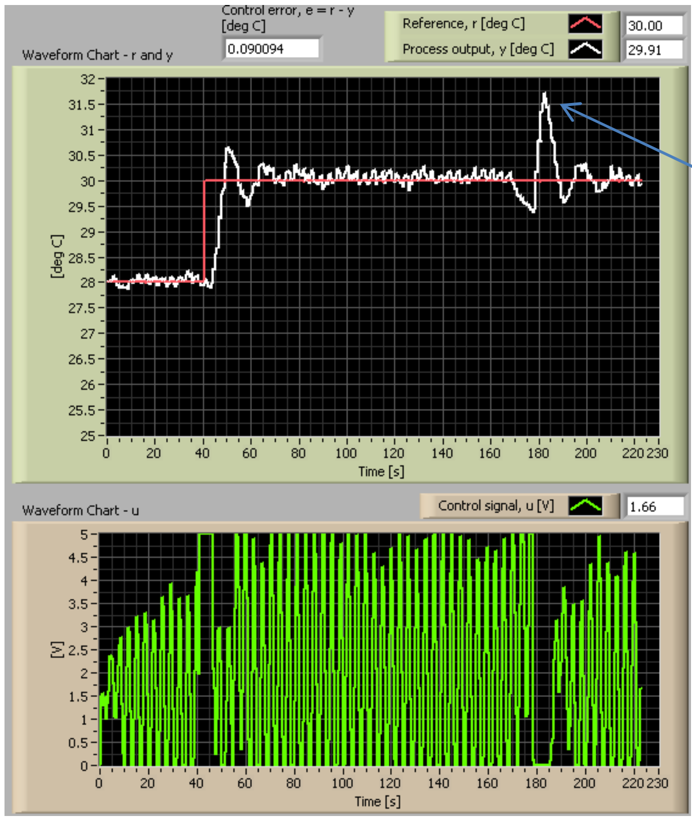
Playing with weight of control signal change (control increment)

Weight increased from 40 to 1000:



Sluggish control!

Weight decreased from 40 to 0.01:



Disturbance applied by covering air inlet with hand for 10 sec)

Fast, but abrupt control!
(”Dead-beat” control)

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Conclusions

- **Reference tracking: MPC starts adjusting the control signal ahead of reference changes, while PID can not start before. MPC gives substantially less control error.**
- **Disturbance (non-measured) compensation: MPC and PID almost equal.**
- **Propagation of measurement noise through controller: Less with MPC than PID**
- **MPC: Constrained control: The controller is able to limit the process output variable (temperature) according to the set constraint.**
- **MPC: Setting a small control horizon: More sluggish and less stable control**
- **MPC: Weight of control signal increment:**
  - **Reducing weight: Fast, but abrupt control. Similar to on/off (”dead-beat”) control**
  - **Increasing weight: Sluggish control**