

Air Rocket Thrust Experiment Involving Computerized Data Acquisition, Calibration, and Uncertainty Analysis

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Abstract

The development and modification of a laboratory experiment to determine the thrust characteristics of an air propelled rocket is described. The experiment is used in the junior level Instrumentations and Measurements course in the Department of Mechanical Engineering at the University of Tulsa. It involves elements of instrument calibration, computerized data acquisition, and uncertainty propagation. The experimental details of the laboratory are described along with the goals of the experiment and a description of the data analysis procedure. Student opinions of the lab experience as determined by a survey about the lab are presented and ways to improve the lab based on student comments are discussed.

Introduction

Three important concepts that are covered in most experimental methods courses are instrument calibration, computerized data acquisition, and propagation of uncertainty into a calculated result. While these concepts are often taught in the classroom, students benefit more by actively learning about these concepts in the laboratory. This paper describes an experiment that has been developed for the Instrumentation and Measurements course in the University of Tulsa's Department of Mechanical Engineering. This junior level lecture-laboratory course contains bi-weekly laboratory exercises where student teams perform hands on experiments and then write up their lab experience and results in a formal technical lab report.

The lively laboratory is particularly well received by the students because it involves the firing of an instrumented ballistic pendulum. The excitement of firing a ballistic air rocket in the laboratory adds to student motivation and helps to retain their attention. An initial version of the experiment was developed by Dr. Denis Zigrang at the University of Tulsa and subsequently modified [1]. The modified experiment involves acquiring data using the computer and data acquisition software and calibrating several standard measurement transducers, e.g. accelerometer, rotary variable differential transformer (RVDT), and diaphragm pressure transducer. In the subsequent lab write up, the students are required to calculate the experimental uncertainty in the calculated impulse based on the elemental uncertainties in the measured signals.

Experiment Description

The purpose of the experiment is to determine the thrust characteristics of an air rocket. This experiment should familiarize students with typical instrument calibration procedures, computerized data acquisition, and the operation of a rotary variable differential transformer (RVDT), unbonded strain gage accelerometer, and pressure transducer. After completing the

experiment, the student should be able to calibrate linear instruments such as pressure transducers and accelerometers. The student should also have gained enough knowledge through the experiment and discussion to explain the operating principles of RVDTs, accelerometers, and diaphragm pressure transducers and the components of a data acquisition system.

In the experiment, a modified fire extinguisher (air rocket) is mounted to the end of a pendulum. An electric solenoid valve is attached at the head of the air rocket. On the rocket is also mounted a 1-D unbonded strain gage accelerometer and a diaphragm pressure transducer. A RVDT is mounted on the pivot of the pendulum to measure the angular position of the pendulum arm. A PC based data acquisition system, consisting of a PCI Bus Data Acquisition Board, screw terminal connector, signal conditioning modules, and LabVIEW software is used to record the signals from the transducers and to fire the rocket by opening the electric valve at the head of rocket. The details of the equipment used in the experiment are listed in Table 1 below.

Table 1. Equipment List.

Item	Manufacturer	Model/Version
1. Modified fire extinguisher	Walter Kidde and Company	F-240
2. Electric solenoid valve	Automatic Switch Company	8215C2
3. 1-D unbonded strain gage accelerometer	Statham	652 (A3)
4. Diaphragm pressure transducer	Bourns	2900
5. RVDT	Schaevitz Sensors	R30D
6. Dial output analog pressure gage	Marsh	34693-3
6. PCI Bus Data Acquisition Board	Keithley Instruments, Inc.	KPCI-3108
7. Screw terminal connector	Keithley Instruments, Inc.	STP-36
8. Signal conditioning modules	Omega Engineering, Inc.	OM3
9. LabVIEW software	National Instruments	Version 6.1

The pendulum assembly is shown in Figure 1. The air rocket (shown in Figure 2) is mounted to the end of the pendulum arm with the firing nozzle oriented in the plane of the pendulum rotation. The pendulum rotates on a pair of ball bearings to minimize frictional losses in the system. The RVDT is connected to the rotating axel and sits in a circular mount on the top of the assembly. The pendulum arm can be locked in an arrangement exactly 45 degrees from the vertical position by sliding a pin in the 45 degree positioning slot.

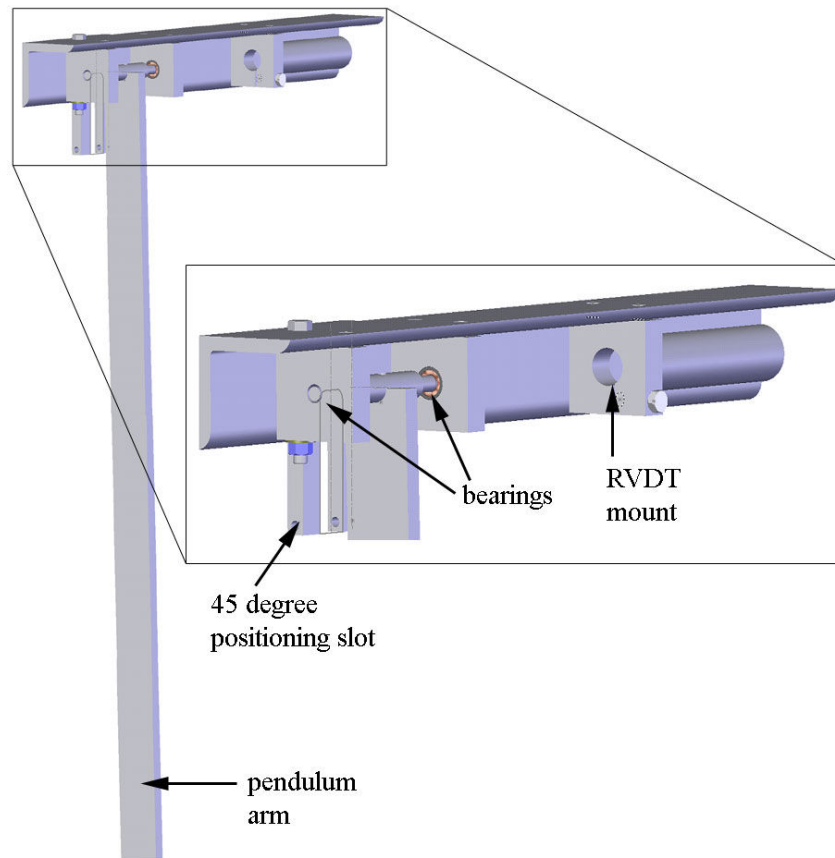


Figure 1. Pendulum Assembly

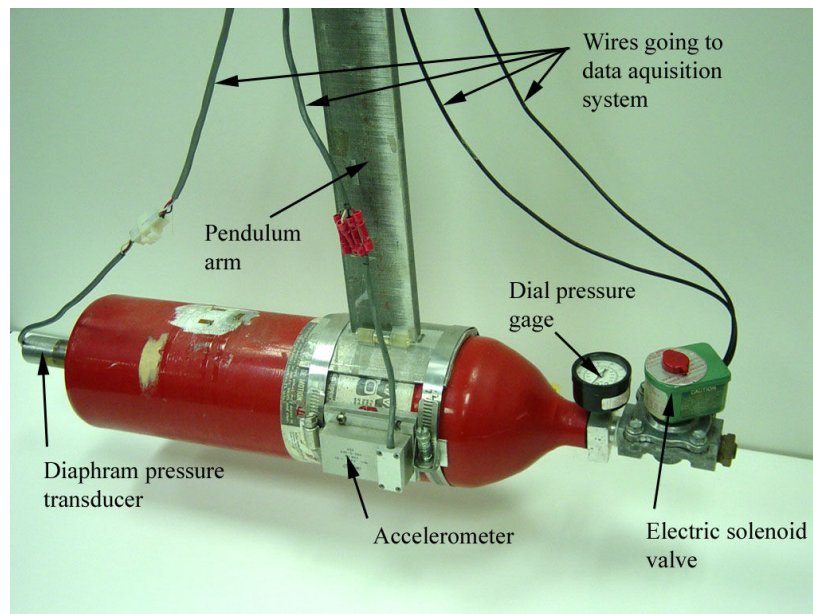


Figure 2. Photograph of Rocket Assembly

The students first calibrate the system by converting the voltage signals from the transducers into pressure, acceleration, and angular position by applying known inputs and adjusting the zero-offset and static sensitivity in the software. For the RVDT and accelerometer, the known inputs are determined by examining the signal while the pendulum is in the vertical position (so that the air rocket is horizontal) and the 45 degree position using the positioning slot. The user interface from the data acquisition software is shown in Figure 3. While the students do not write the data acquisition code or design the front panel user interface, they are shown the underlying LabVIEW wiring diagram and given an explanation of how the code works.

When the rocket is at rest in the horizontal position, the students adjust the zero offset voltage from the RVDT and accelerometer signal so that the effective signals are zero. Then the students position the rocket at 45 degrees. The signal in degrees or acceleration is equal to the mV input multiplied by the static sensitivity of the transducer. The students adjust the static sensitivity for the RVDT on the front panel until the signal reads 45 degrees. Then the students adjust the static sensitivity for the accelerometer until the signal reads $g \cdot \sin \theta = 9.807 \cdot \sin 45^\circ = 6.934 \text{ m/s}^2$. The diaphragm pressure transducer is calibrated by comparing its output to a dial output pressure gage that is connected to the solenoid valve and adjusting its static sensitivity to match the dial gage output.

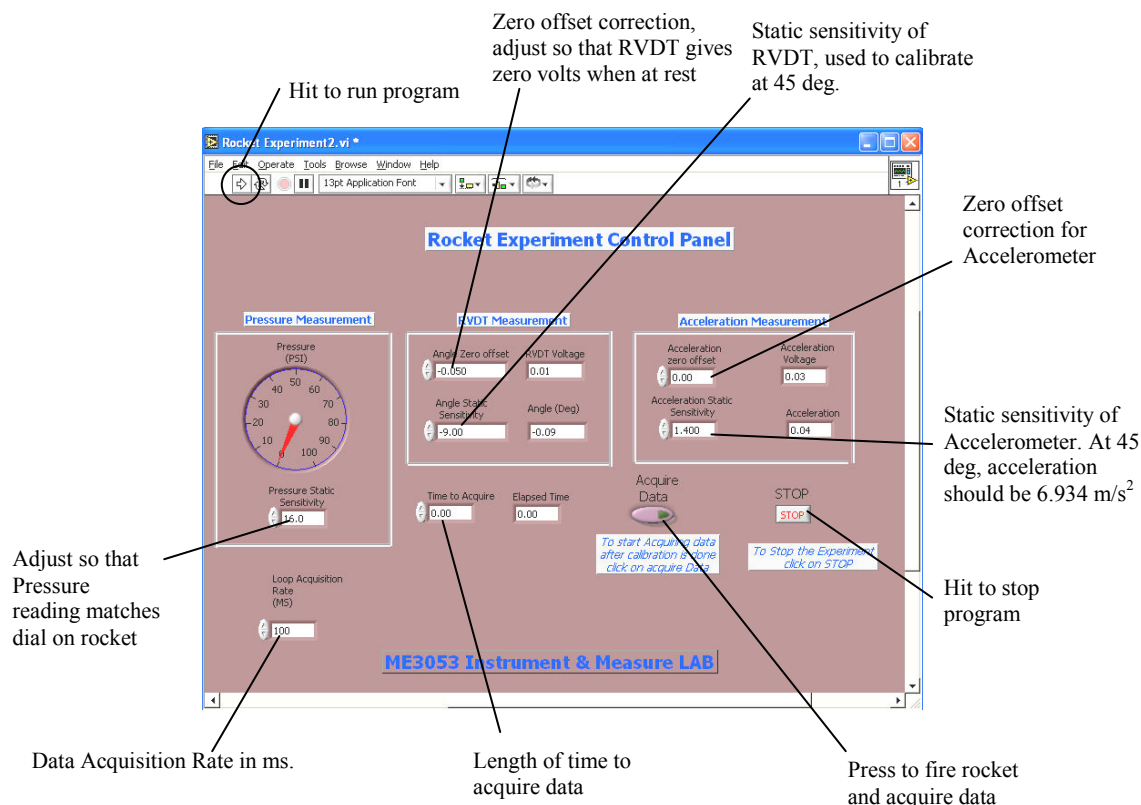


Figure 3. LabVIEW Front Panel

After calibration is complete, the students adjust the data acquisition rate, enter the amount of time that they want to collect data, and send an output voltage from the computer to the valve

attached to the air rocket, opening the valve and allowing air to escape through a nozzle. The students perform four launches of the air rocket assembly, with initial pressures of 20, 40, 60, and 80 psi.

The students also determine the natural period of oscillation by raising and releasing the pendulum and recording the RVDT output. By recording and reviewing the RVDT output, the students can easily find the period of oscillation which is used to calculate the polar moment of inertia in the analysis.

Analysis Description

The following nomenclature is used in the analysis of the data

- T - Time period of oscillations (s)
- g - Acceleration due to gravity (9.807 m/s^2)
- m_t - Total mass = $m_R + m_L$ = mass of rocket + mass of leg (kg)
- L_{cg} - Length from pivot to center of gravity of rocket assembly (m)
- J - Polar moment of inertia of rocket assembly (kg-m^2)
- J_L - Polar moment of inertia of leg (kg-m^2)
- J_R - Polar moment of inertia of rocket (kg-m^2)
- L_1 - Length of leg (m)
- L_2 - Length of weld filling (m)
- L_3 - Length of rocket (vertical length = external diameter of rocket), (m)
- L_f - Length from pivot to the central axis of rocket (m)
- F - Force exerted by the rocket by release of compressed air (N)
- θ - Angle between the leg and the vertical line at a particular instant (deg.)
- I - Impulse (N·s)
- a - Acceleration experienced by the rocket at a particular instant (m/s^2)
- α - Angular acceleration experienced by the rocket at a particular instant (rad/s^2)

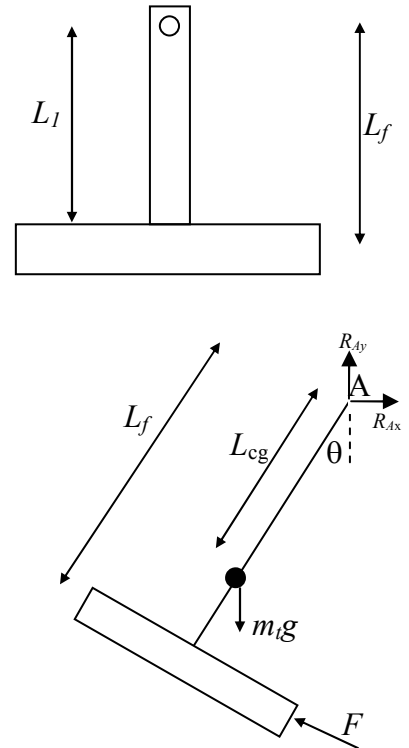


Figure 4. Geometry and Free Body Diagram

In the analysis, the students first compute the polar moment of inertia of the pendulum rocket assembly using the period of oscillation method and by direct calculation.

Computation of Polar Moment of Inertia, Period of Oscillation method:

$$T = 2\pi \sqrt{\frac{J}{g m_t L_{cg}}} \quad \rightarrow \quad J = \frac{m_t g L_{cg} T^2}{4\pi^2} \quad \text{eq. (1)}$$

Computation of Polar Moment of Inertia, Direct Calculation:

$$J = J_L + J_R \quad \text{eq. (2)}$$

$$J_L = \int_0^{L_1} x^2 \left(\frac{m_L}{L_1} \right) dx = \frac{m_L L_1^2}{3} \quad \text{eq. (3)}$$

$$J_R = m_R L_f^2 \quad \text{eq. (4)}$$

$$J = \frac{m_L L_1^2}{3} + m_R L_f^2 \quad \text{eq. (5)}$$

Computation of thrust, F, as a function of time and Impulse, I:

Next they compute the thrust as a function of time in terms of the measured acceleration and angular position. They begin by summing the moments about the pivot of the pendulum, point A.

$$\Sigma M_A = FL_f - m_t g L_{cg} \sin \theta = J\alpha \quad \text{where } \alpha = \frac{a - g \sin \theta}{L_f} \quad \text{eq. (6)}$$

$$FL_f - m_t g L_{cg} \sin \theta = \frac{J}{L_f} (a - g \sin \theta) \quad \text{eq. (7)}$$

$$F = \frac{J}{L_f^2} (a - g \sin \theta) + m_t g \left(\frac{L_{cg}}{L_f} \right) \sin \theta \quad \text{eq. (8)}$$

F is implicitly dependent on time. Next the students calculate the total impulse of the rocket for each initial pressure by solving $I = \int F dt$ using numerical integration.

Experimental Error Analysis:

Finally, the students calculate the uncertainty in the impulse and thrust by propagating the uncertainty from the individual transducer and dimensional measurements into the results. The propagation of uncertainty in the variables to the result yields an uncertainty estimate by

$$u_f = \sqrt{\sum_{i=1}^n \left(\frac{\partial F}{\partial X_i} \cdot u_{x_i} \right)^2} \quad (95\%) \quad \text{eq. (9)}$$

where each X_i is a variable used to calculate the thrust, F , and u_{x_i} is the estimated 95% uncertainty in each of these variables [2]. Assuming negligible uncertainty in the gravitation constant, g , there are six variables that contribute to the thrust in equation 8. Combining equation 8 and 9 yields

$$u_f = \sqrt{\left(\frac{\partial F}{\partial J} u_J\right)^2 + \left(\frac{\partial F}{\partial L_f} u_{L_f}\right)^2 + \left(\frac{\partial F}{\partial a} u_a\right)^2 + \left(\frac{\partial F}{\partial m_t} u_{m_t}\right)^2 + \left(\frac{\partial F}{\partial L_{cg}} u_{L_{cg}}\right)^2 + \left(\frac{\partial F}{\partial \theta} u_\theta\right)^2} \quad \text{eq. (10).}$$

The uncertainty in the polar moment of inertia, u_J , and uncertainty in the length from the pivot to the thrust line, u_F , are given below.

$$u_J = \sqrt{\left(\frac{\partial J}{\partial m_t} u_{m_t}\right)^2 + \left(\frac{\partial J}{\partial L_{cg}} u_{L_{cg}}\right)^2 + \left(\frac{\partial J}{\partial T} u_T\right)^2} \quad \text{eq. (11)}$$

$$u_{L_f} = \sqrt{\left(\frac{\partial L_f}{\partial L_1} u_{L_1}\right)^2 + \left(\frac{\partial L_f}{\partial L_2} u_{L_2}\right)^2 + \left(\frac{\partial L_f}{\partial L_3} u_{L_3}\right)^2} \quad \text{eq. (12)}$$

The partial derivatives in equation 10–12 are determined from equation 8 and listed below:

$$\frac{\partial F}{\partial J} = \frac{a - g \sin \theta}{L_f^2} \quad \text{eq. (13)}$$

$$\frac{\partial F}{\partial L_{cg}} = \frac{m_t g \sin \theta}{L_f} \quad \text{eq. (14)}$$

$$\frac{\partial F}{\partial a} = \frac{J}{L_f^2} \quad \text{eq. (15)}$$

$$\frac{\partial F}{\partial m_t} = g \sin \theta \left(\frac{L_{cg}}{L_f} \right) \quad \text{eq. (16)}$$

$$\frac{\partial F}{\partial L_f} = \frac{-2J(a - g \sin \theta)}{L_f^3} - m_t g \sin \theta \left(\frac{L_{cg}}{L_f^2} \right) \quad \text{eq. (17)}$$

$$\frac{\partial F}{\partial \theta} = \frac{-Jg \cos \theta}{L_f^2} + \frac{m_t g \cos \theta L_{cg}}{L_f} \quad \text{eq. (18)}$$

The uncertainty in the force and impulse are written in terms of percentage below:

$$\%U_F = (U_F / F) * 100\% \quad \text{eq. (19)}$$

$$\%U_I \cong \%U_F \text{ (Ignoring } U_t) \quad \text{eq. (20)}$$

Going from equation 19 to equation 20 is not entirely correct because the uncertainty in F is not a constant value, but changes with time, while the uncertainty in impulse should be a fixed value. Students can choose to use the percent uncertainty in F at the maximum thrust to estimate the percent uncertainty in impulse from equation 20, or use more detailed uncertainty analysis to propagate the uncertainty through the numerical integration.

Typical Results

After firing the rocket at the four different pressure levels and recording the data, the students plot the measured variables (angular position, pressure, and acceleration) versus time using Excel. Then with their Excel data they calculate the thrust versus time using equation 8. Typical results at an initial pressure of 80 psi are shown in Figure 5.

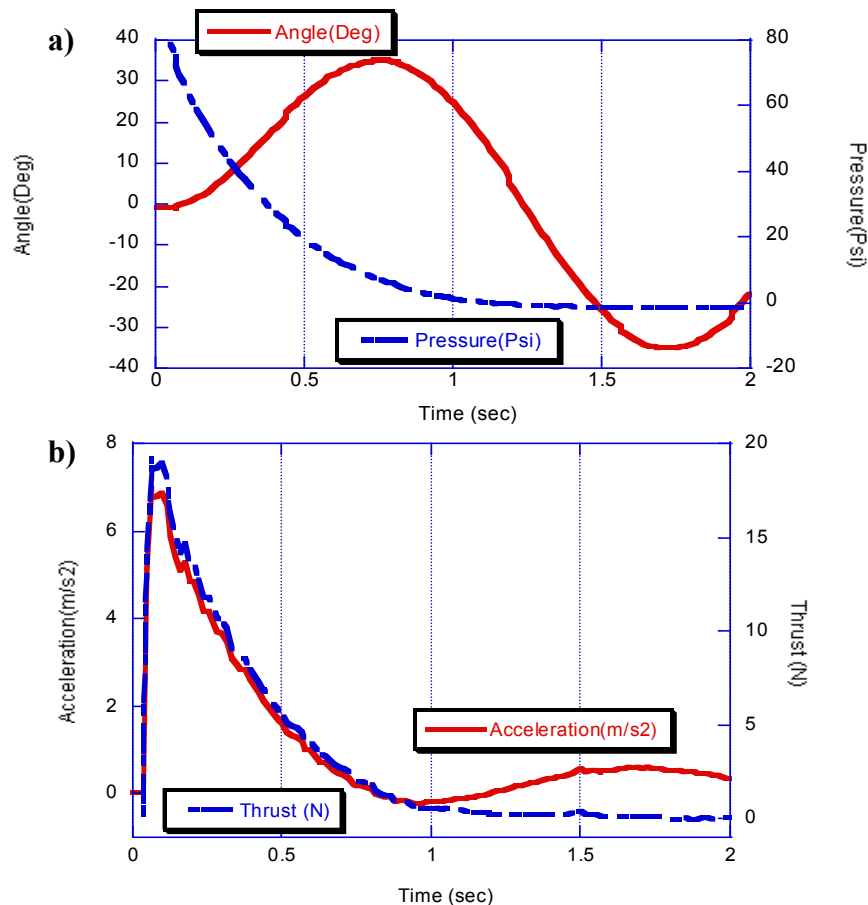


Figure 5. Typical results from the experiment (a) Angular displacement and pressure versus time for initial pressure of 80 psi. (b) Acceleration and calculated thrust versus time for initial pressure of 80 psi.

Next, the students calculate the impulse and associated uncertainty in the force and impulse as a function of time. Typical results for the four different initial pressures are shown in Figure 6. The error bars in Figure 6 represent the U_{95} uncertainty estimated with equation 20.

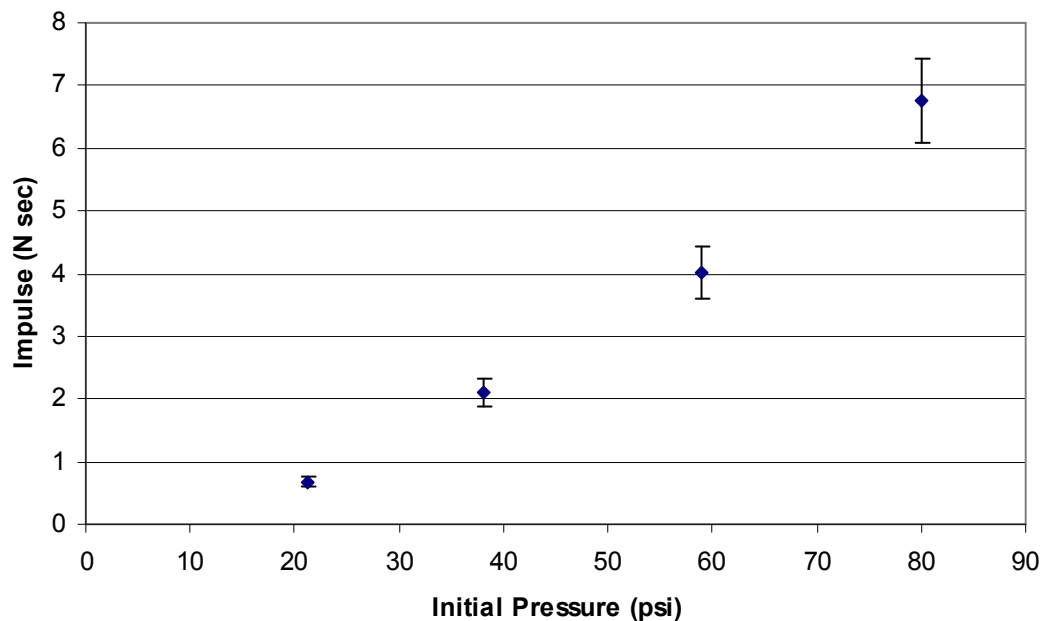


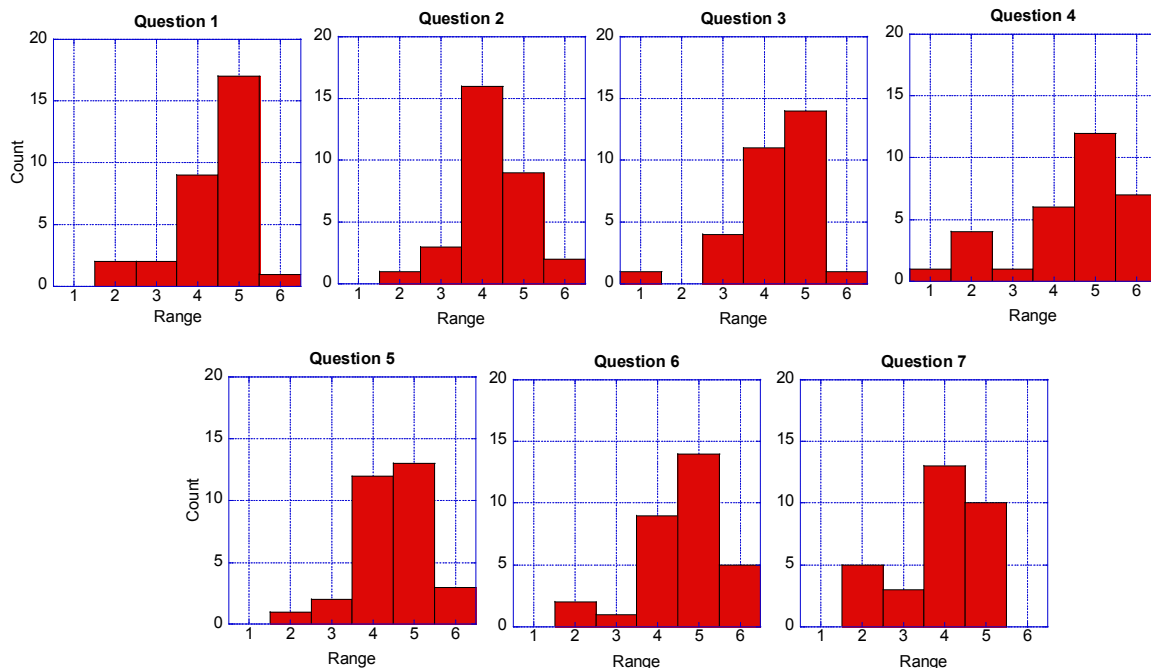
Figure 6. Typical results of calculated impulse and associated uncertainty for four different initial pressures.

Student Assessment

After the students turned in their laboratory reports (two weeks after performing the experiments) a short survey was given in order to assess the effectiveness of and student attitudes towards this laboratory exercise. The survey contained nine questions/comments. The first seven comments used a six point Likert scale to indicate if they strongly disagree, disagree, slightly disagree, slightly agree, agree, or strongly agree with each statement. A number from 1 (for strongly disagree) to 6 (for strongly agree) was assigned to each selection. The last two questions were open ended short answer questions asking the students to list strengths of the Rocket Engine Thrust Experiment and comment on how the experiment could be improved. A total of 31 students responded to the survey. The first seven questions from the survey are listed in Table 2 below. Histograms, showing the distribution of Likert scale responses for each question, are shown in Figure 7.

Table 2. Likert scale questions from survey.

Question/Comment	Average Response
1. The experiment did a good job of familiarizing me with typical instrument calibration procedures.	4.42
2. The laboratory did a good job of familiarizing me with computerized data acquisition and LabVIEW.	4.26
3. The laboratory did a good job of familiarizing me with the operation of a rotary variable differential transformer (RVDT), accelerometer, and pressure transducer.	4.29
4. Detailed uncertainty analysis became more clear as a result of the experiment and lab write-up.	4.45
5. The lab and subsequent lab report caused me to think critically about sources of experimental error that contribute to uncertainty in the rocket engine thrust.	4.48
6. Overall the lab was a positive experience.	4.61
7. Overall the lab write-up was a positive experience.	3.90

**Figure 7. Histogram results for Likert scale results for comments 1 through 7. Strongly Disagree = 1, Strongly Agree = 6.**

Some of the comments from the first open ended question about the strengths of the experiment are listed below.

- It was fun to see how the set up worked
- The uncertainty analysis calculations become more clear, and were a good compliment to the material studied in Ch. 5.

- Being able to see how the calibration of pressure had to be adjusted really helped me see how error changes through the measurement
- I enjoyed learning about the data acquisition system
- Good set of data collected, fairly easy to analyze
- Calibration, data acquisition, and LabVIEW understanding
- I got exposed to partial derivative and uncertainty more
- Performing the uncertainty analysis in the lab write up helped me understand how uncertainty propagates
- Clarity in how to interpret the data, calculate thrust and impulse, and find various uncertainties. Helped me better understand the effect of uncertainties in measurement
- It was cool.
- It made you think of error that occurred. It also made you realize how important calibration through an experiment is.
- One of the greatest strengths of the experiment was that we were forced to find uncertainty as a function of changing variables and therefore saw exactly how uncertainty can change with time for an experience.
- It was fun.

Examples of comments about how the lab could be improved include the following.

- I thought the lab was great. The only problem I had was with the write up. I guess if the calculations in the write up could have been further explained that would improve the lab.
- I found the amount of data collected and put in excel was very cumbersome and tedious.
- If we could learn how to use the program prior to using it in the lab.
- Step through an example of calculating the uncertainty analysis.
- Loose the annoying TA, more student involvement.
- None, good as it is.
- Don't really have any problem with it
- I was lost when LabVIEW was being explained since it was the first time I saw it.
- Further explanation of the program and components (RVDT, transducer, accelerometer) used to gather the data.
- More background on how the pressure transducer actually works.
- Not sure it can. It seemed very tedious
- Learn more about LabVIEW
- I would like to have programmed LabVIEW on my own to calibrate the transducers.

Conclusions

Improvements to the laboratory can certainly be made based on some of these student comments. One suggestion is to lengthen the lab by having the students program the data acquisition code themselves. Also, by spending more time (or more class time before the lab) discussing the operating principles of RVDTs, accelerometers, and diaphragm pressure transducers the lab experience would be strengthened. However, for the most part, this laboratory exercise worked well to familiarize the students with typical instrument calibration procedures, computerized data acquisition, and detailed uncertainty analysis. Over 90% of the students at least partially agreed that the laboratory was a positive experience based on the post experiment survey.

Appendix

The following values and uncertainties were used in the calculations

- Total Mass $m_t = 3915.4\text{g} \pm 0.1\text{g}$
- Mass of Leg $m_L = 1640\text{g} \pm 0.1\text{g}$
- Center of Gravity : $L_{cg} = 750.5\text{mm} \pm 0.5$
- $L_1 = 910.5\text{mm} \pm 0.5\text{mm}$
- $L_2 = 4.97\text{mm} \pm 0.05\text{mm}$
- $L_3 = 115.0\text{mm} \pm 0.5\text{mm}$
- $L_f = L_1 + L_2 + L_3/2$
- Gravitational acceleration $g = 9.807\text{ m/s}^2$ assume exact.
- $u_a = \pm 0.03\text{m/s}^2$
- $u_\theta = \pm 0.5^\circ$
- $u_T = \pm 0.5\text{ sec.}$

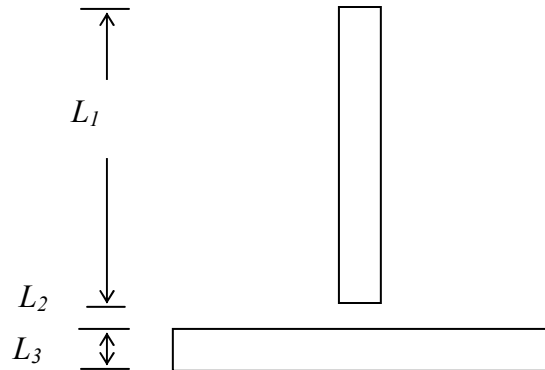


Figure 8. Definition of L_1 , L_2 and L_3

References

- [1] Zigrang, Denis J. *Elements of Engineering Measurements*, Revision 6, College of Engineering, University of Tulsa. August, 1989.
- [2] Figliola, Richard S. and Beasley, Donald E. *Theory and Design for Mechanical Measurements*, 3rd ed. New York: John Wiley & Sons, Inc., 2002

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